air pollution training institute

4.

introduction to environmental statistics

self-instructional course

SI 473

MODULE IV

ANALYSIS OF VARIANCE

UNITED STATES ENVIRONMENTAL PROTECTION AGENCY

ANALYSIS OF VARIANCE (ANOVA)

1

On occasion you may be presented with data on several samples, and want to know if all the samples could have been drawn from populations having equal means. In other words, you want to determine if

$$H_0: \mu_1 = \mu_2 = \mu_3 = \ldots \mu_n$$

or if

$$H_1: \text{not all } \mu\text{'s are equal}$$

50

Duncan's Multiple Range test can be used to test differences among sample means once a significant F value has been obtained in the ANOVA. If a significant F in the ANOVA is not found, you should stop your testing at that point. Before continuing with a Duncan Test, you should be certain that your obtained F in the ANOVA is (greater than/less than) the critical value.

From what we know already, one method to make this determination could be to test all possible combinations, using the two-sample *t* test. If you had four samples, you could propose testing each of the following individually:

$$H_o: \mu_1 = \mu_2$$

$$H_o: \mu_1 = \mu_3$$

$$H_o: \mu_1 = \mu_4$$

$$H_o: \mu_2 = \mu_3$$

$$H_o: \mu_2 = \mu_4$$

$$H_o: \mu_3 = \mu_4$$

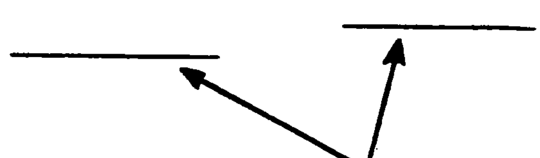
50a

Answer: greater than

51

Duncan's procedure makes use of an ANOVA summary table and Table H in the Guide. It does, however, require equal sample sizes. The final outcome of this procedure is a table that looks like this:

	$\bar{x}_1$	$\bar{x}_2$	$\bar{x}_3$	$\bar{x}_4$
$\bar{x}_1$				
$\bar{x}_2$				
$\bar{x}_3$				



And any means not underscored by the same lines are considered significantly different. This will be explained in more detail a little later.

This is clearly a long way to determine the composite null hypothesis. In fact, this procedure is statistically inappropriate. The appropriate technique to use is the analysis of variance (ANOVA). In ANOVA, you partition the total variability of all four samples into the variability *between* the samples and the variability *within* samples. Then, by comparing these partitioned variabilities, you can determine if all of the means are indeed equal. This will become clearer as you read further.

Duncan's test consists of 5 steps that are outlined here and presented in more detail using an example in subsequent frames.

Step A: Rank order the sample means.

Step B: Find the *standard error* of the mean.

Step C: Determine the *shortest significant ranges*.

Step D: Construct a table of differences between means.

Step E: Compare the differences between means with the shortest significant ranges.

A standard error of any statistic expresses how much variation can be expected if that statistic is calculated on many samples of size n drawn from the same population. Here we are interested in the standard error of the mean.

This module will emphasize the following:

1. The total variability for the group of samples is the sum of the variability *between* samples and the variability *within* samples.

2. Through certain statistical computations you can partition this total variability into two components: variability *between* samples and variability *within* samples.

3. Through an F test on the *between* and *within* variances, you can determine, with a predetermined degree of confidence, whether all the samples were drawn from the same population.

Step A: The first step of Duncan's procedure is to rank order the means we wish to test. Let's consider our initial ANOVA problem, where we had an F of 38 and the following data:

$\bar{x}_1 = 14$ $\bar{x}_2 = 15$ $\bar{x}_3 = 22$

Source	SS	df	MS	F
Between	190	2	95	38
Within	30	12	2.5	
Total	220	14		

Rank order the means from lowest to highest.

______

______

______

5

Now, read Chapter 18, pp. 221-229 (up to 18-4) in the text, and then go to frame 6.

53a

Answer: 14

15

22

54

Step B: Next we find the *standard error* of the mean by using the following formula:

$$s_{\bar{x}} = \frac{s}{\sqrt{n}}$$

where $s_{\bar{x}}$ = the standard error

s = the square root of MS_{within}

n = the number of values on which each mean is based

Let's fill in these values.

$$s = \sqrt{MS_{within}}$$

$$= \sqrt{\#\_\_\_\_\_}$$

$$n = \#\_\_\_\_\_$$

6

In the analysis of variance (ANOVA) the statistician tests the hypothesis

H_o: ____________

against the alternative hypothesis

H_1: ____________

54a

Answer: $s = \sqrt{2.5}$

$s = 1.58$

$n = 5$

55

Let's calculate $s_{\bar{x}}$.

$$s_{\bar{x}} = \frac{s}{\sqrt{n}}$$

$$= \frac{1.58}{\sqrt{5}}$$

= # ________

6a

Answer: H_o: $\mu_1 = \mu_2 = \mu_3 = \ldots \mu_n$

H_1: not all μ's are equal

7

To do this, you partition the variability of the samples into two components:

the variability between samples

and

the variability within samples.

When added, these result in the total variability.

Total variability = * ______________ + * ______________

55a

Answer: $s_{\bar{x}} = \frac{1.58}{2.24}$

$s_{\bar{x}} = .705$

56

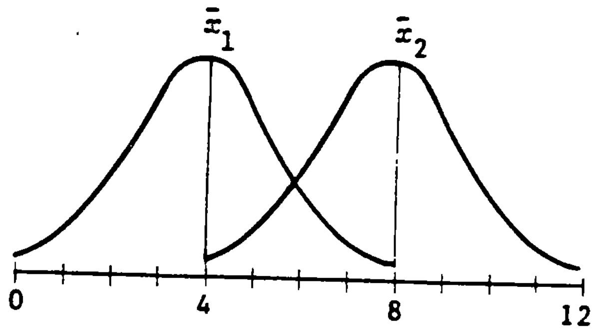
Step C: Now we need to find the *shortest significant ranges*. To do this turn to Table H in the Guide. This table lists the Significant Studentized Ranges for Duncan's New Multiple Range Test with $\alpha = .05$ and .01. Let's use .05. Enter the table at the row with degrees of freedom equal to df_w. In this case, looking back at the ANOVA table (see frame 53 p. IV-4), df_w = # ______.

7a

Answer: the variability between samples + the variability within samples.

8

Let's look at this relationship graphically for a two-sample situation. Assume we have the following distributions when the data are plotted.



What are the values of $\bar{x}_1$ = # ______

$\bar{x}_2$ = # ______

56a

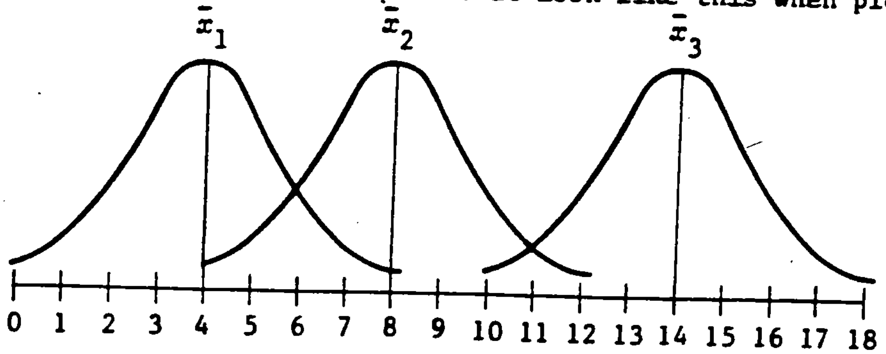
Answer: 12

57

k is the number of means we are comparing.

k = # ______

Assume now that we have three samples that look like this when plotted: 11



From the graph above, determine the values of $\bar{x}_1$, $\bar{x}_2$, and $\bar{x}_3$.

$\bar{x}_1$ = # ______ $\bar{x}_2$ = # ______ $\bar{x}_3$ = # ______

59a

Answer: $R_3 = 2.273$

$R_2 = 2.172$

60

These values are the *shortest significant ranges.*

Answer: $\bar{x}_1 = 4$

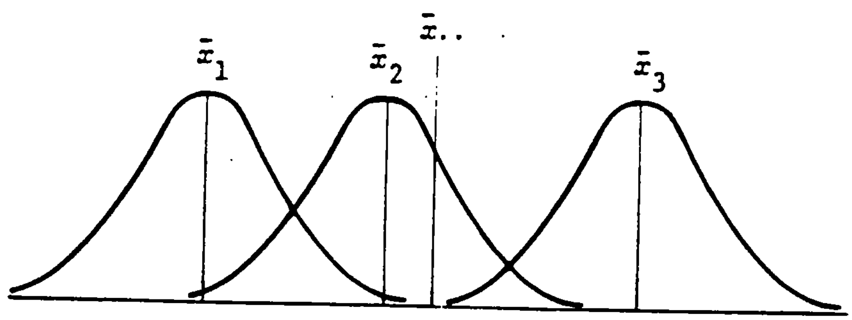
$\bar{x}_2 = 8$

$\bar{x}_3 = 14$

Pretty easy, huh?

12

In the same distributions we are now going to introduce a new term to you--the *grand mean*. The grand mean is simply the mean you would obtain if you treated the three samples as one sample and computed the mean.



The grand mean is symbolized by ______.

61

Step D: Next we construct a table, arranging the means from lowest to highest.

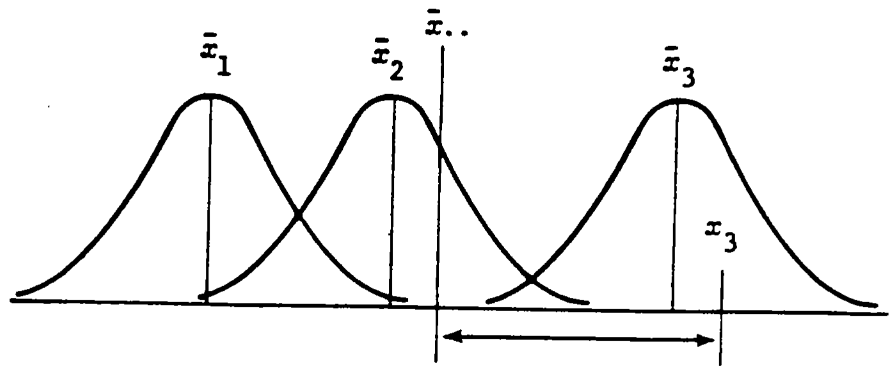
	Means	$\bar{x}_1$ 14	$\bar{x}_2$ 15	$\bar{x}_3$ 22	Shortest Significant Ranges
$\bar{x}_1$	14		1	8	$R_2 = 2.172$
$\bar{x}_2$	15			7	$R_3 = 2.273$
$\bar{x}_3$	22				

The entries in the table were derived by subtracting the means (15-14 = 1), (22-14 = 8), and (22-15 = 7).

Answer: $\bar{\bar{x}}..$

13

Let's go back to the graphic representation of the variabilities.

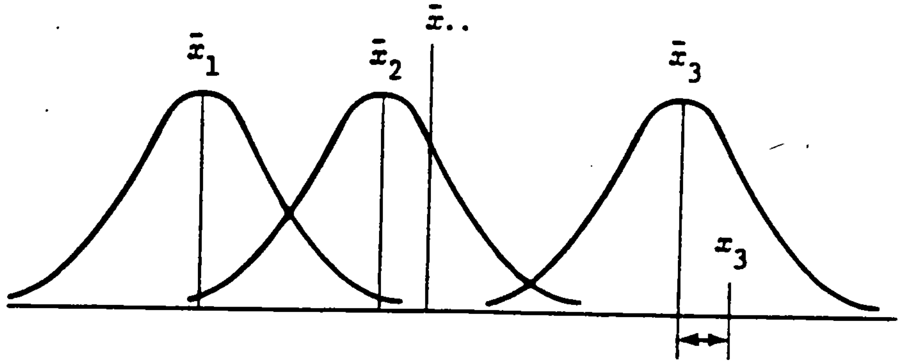


The deviation of each individual value from the grand mean contributes to *total* variability (as represented above by the deviation or distance from the data value x_3 to the grand mean).

62

Step E: Now we compare the difference between the largest mean and the smallest mean with the appropriate shortest significant range. Because $\bar{x}_3 - \bar{x}_1$ is the range of three means, the difference between them must exceed R_3 in order to be a significant difference. In this case, since 8 is larger than 2.273, this difference is considered __________.

14



The deviation between each individual score and the mean of the sample to which that score belongs contributes to variability *within* samples.

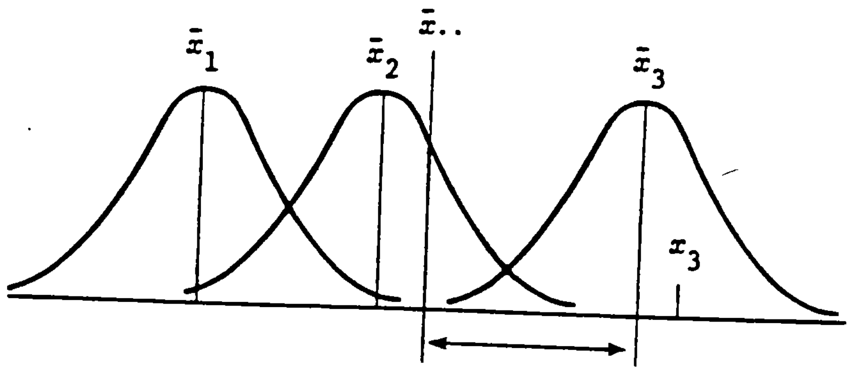
62a

Answer: significant

63

The next comparison is made between the largest mean and the second smallest. In this case

$$\bar{x}_3 - \bar{x}_2 = \#\_\_\_\_\_\_\_\_$$



The deviation between each sample mean and the grand mean contributes to variability *between* samples.

63a

Answer: 7

64

This difference is tested against R_2 since it covers a range of #_______ means.

16

The beauty of the analysis of variance (ANOVA) is that the deviation of a score from the grand mean equals the deviation of that score from the mean of the sample to which it belongs plus the deviation of the sample mean from the grand mean.

64a

Answer: 2

65

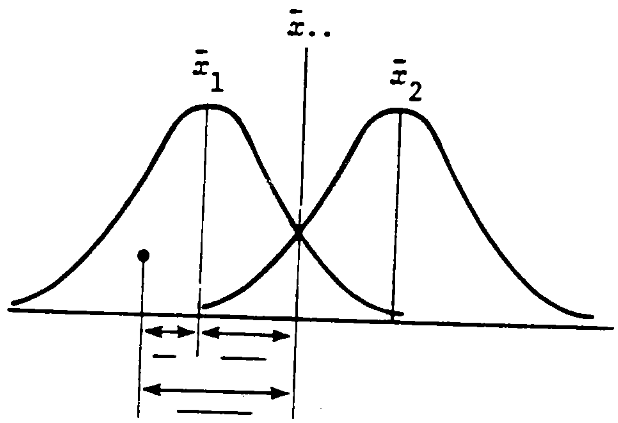
Since 7 is larger than 2.172, this difference is ____________.

Label the following deviations:

B for between

W for within

T for total



65a

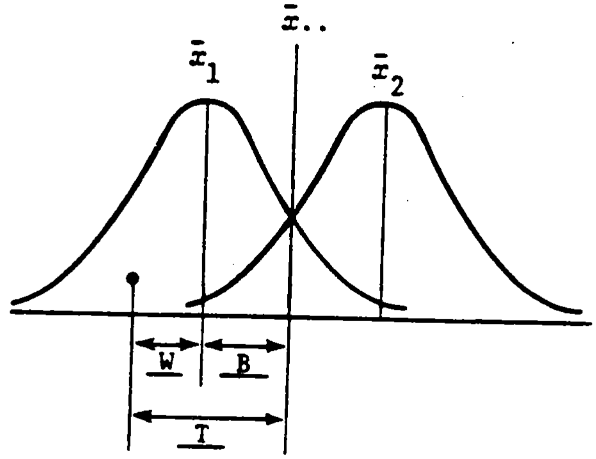
Answer: significant

66

Since we are finished comparing the largest mean with all others, we move on to compare the second largest minus the smallest.

$\bar{x}_2 - \bar{x}_1 = \#$ ______

Answer: 17a



Answer: 1 66a

67

This difference is again compared with R_2 since it covers a range of 2 means.

Is this difference significant? ________ (yes/no)

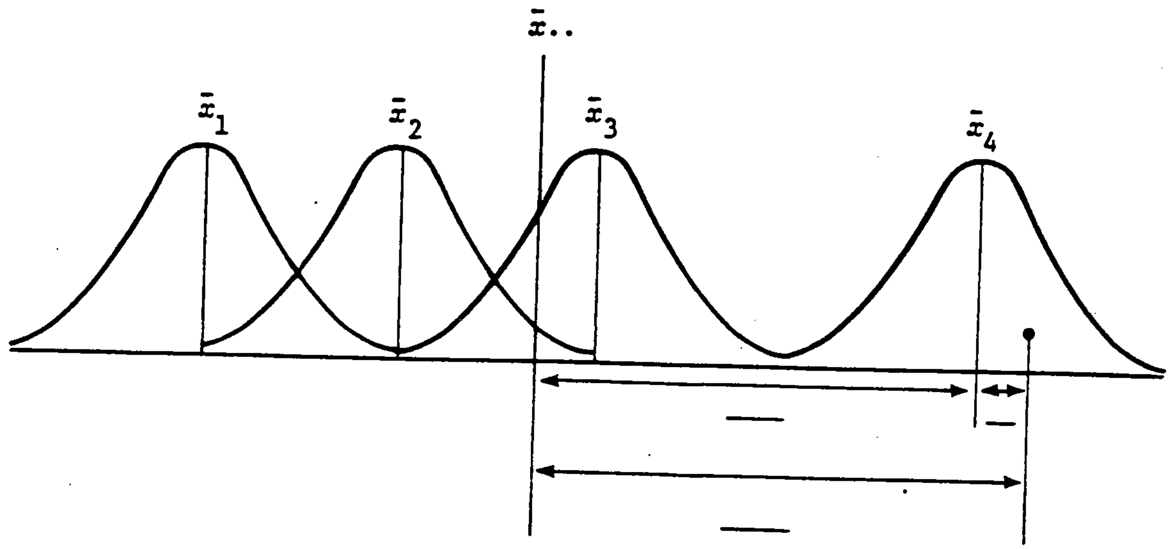
Why? †________

Label the following:

B for between

W for within

T for total



67a

Answer: No

The difference is not larger than R_2.

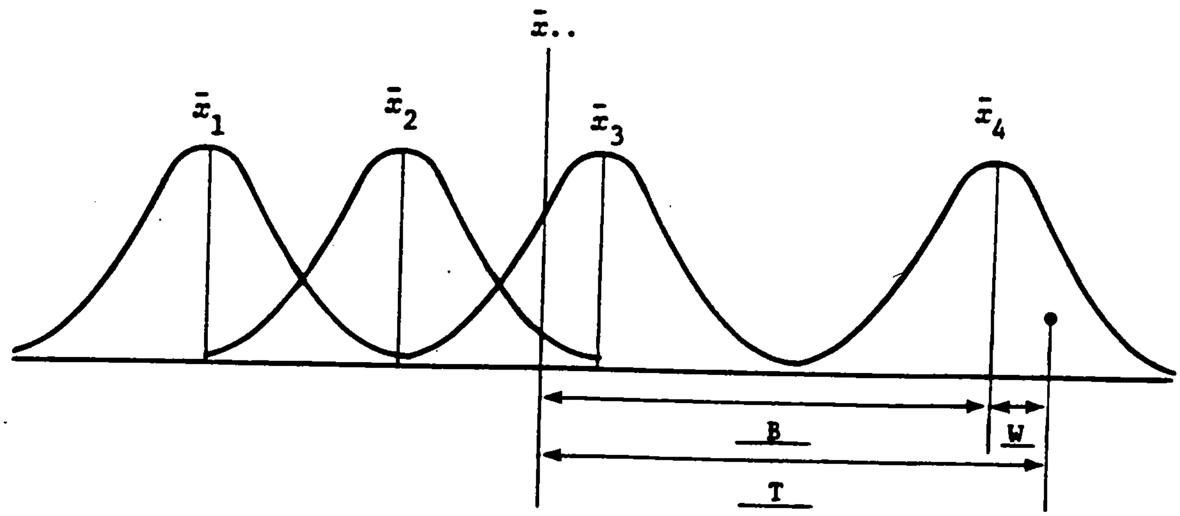
68

To summarize, then:

$\mu_1 = \mu_2$

$\mu_1 \neq \mu_3$ (since the difference was found to be significant)

$\mu_2 \neq \mu_3$ (again because the difference was significant)



19

When all the deviations mentioned (within, between, total) for all values are squared and summed, we obtain three statistics called:

Sums of Squares Total (SS_T)
Sums of Squares Within (SS_W)
Sums of Squares Between (SS_B)

69

Let's try one that's a bit more difficult. Assume you have calculated the following:

$\bar{x}_1 = 36.4$ $\bar{x}_2 = 39.7$ $\bar{x}_3 = 46.0$ $n = 4$

$\bar{x}_4 = 57.3$ $\bar{x}_5 = 62.6$ $\bar{x}_6 = 69.4$

Source	SS	df	MS	F
Between	469.6	5	93.92	5.87
Within	288.0	18	16.00	
Total	757.6	23		

We want to know which differences between means are significant. (Use $\alpha = .05$)

20

If you knew the values of SS_B and SS_W, you could easily find SS_T by simply adding $SS_B + SS_W$.

SS_T = _____ + _____

70

Step A: First, rank order the means. (We've already done this for you.)

Step B: Next, find $s_{\bar{x}}$

$s_{\bar{x}} = \frac{s}{\sqrt{n}}$ Remember: $s = \sqrt{MS_w}$

$s_{\bar{x}}$ = _____

Answer:

$$SS_T = SS_B + SS_W$$

21

Let's now see how to compute the analysis of variance (from now on we will call it ANOVA). To compute ANOVA, we need to know two of three sums of squares. The easiest to compute are SS_T and SS_B. Since SS_B and SS_W are additive, we can then find SS_W by (addition/multiplication/division/subtraction) ($SS_W = SS_T - SS_B$).

70a

Answer: $s_{\bar{x}} = \frac{4}{\sqrt{4}}$

$= \frac{4}{2}$

$= 2$

71

Now turn to Table H and find the values for # _____ df and k = # _____

Answer: subtraction

22

First let's look at the formula we will be using to find SS_T.

$$SS_T = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{N}$$

(Note the similarity to the numerator of:

$$s^2 = \frac{\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}}{n - 1}$$
)

This is calculated the same way as the variance. (We use N in place of the text's kn. N = total number of values in all samples, and will equal kn when each sample is the same size. The use of kn does not consider the possibility that all the samples may not be the same size, and equal n's are not required for ANOVA.)

71a

Answer: $df = df_w = 18$

$k = 6$ (since we are testing 6 means)

The values listed are 2.971, 3.118, 3.210, 3.274, 3.321

72

Step C: Find the shortest significant ranges.

shortest significant range = $\left(s_{\bar{x}}\right)$ (tabled values)

R_2 = #______

R_3 = #______

R_4 = #______

R_5 = #______

R_6 = #______

Given the following values, we will compute SS_T.

$$SS_T = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{N}$$

x_1	x_2	x_3
12	13	20
13	14	21
14	15	22
15	16	23
16	17	24
70	75	110

Go on to Frame 24.

Answer:

$R_2 = (2.971)\ (2) = 5.942$

$R_3 = (3.118)\ (2) = 6.236$

$R_4 = (3.210)\ (2) = 6.420$

$R_5 = (3.274)\ (2) = 6.548$

$R_6 = (3.321)\ (2) = 6.642$

24

x_1	x_1^2	x_2	x_2^2	x_3	x_3^2
12	144	13	169	20	400
13	169	14	196	21	441
14	196	15	225	22	484
15	225	16	256	23	529
16	256	17	289	24	576
	990		1135		2430

Step A: Find Σx_i^2. Square all values and add them.

Σx_i^2 = # ____________

73

Step D: Fill in the blanks in the following table:

	$\bar{x}_1$ 36.4	$\bar{x}_2$ 39.7	$\bar{x}_3$ 46.0	$\bar{x}_4$ 57.3	$\bar{x}_5$ 62.6	$\bar{x}_6$ 69.4	Shortest Significant Ranges
$\bar{x}_1$ 36.4		3.3	____	20.9	26.2	____	R_2 = #_____
$\bar{x}_2$ 39.7			6.3	____	____	29.7	R_3 = #_____
$\bar{x}_3$ 46.0				11.3	____	23.4	R_4 = #_____
$\bar{x}_4$ 57.3					5.3	____	R_5 = #_____
$\bar{x}_5$ 62.6						____	R_6 = #_____
$\bar{x}_6$ 69.4							

For answer, fold out frame 1000 at the end of this module and then go to Frame 74, page IV-27.

Answer: $990 + 1135 + 2430 = 4555$ 24a

$\Sigma x_i^2 = 4555$

25

x_1	x_2	x_3
12	13	20
13	14	21
14	15	22
15	16	23
16	17	24
70	75	110

Step B: Find Σx_i.

Add all values.

Σx_i = ________.

Answer: $70 + 75 + 110 = 255$

26

x_1	x_2	x_3
12	13	20
13	14	21
14	15	22
15	16	23
16	17	24
70	75	110

Step C: Find $\frac{(\Sigma x_i)^2}{N}$

$\Sigma x_i = 255$ (from Step B)

N = total number of values of all samples

$\frac{(\Sigma x_i)^2}{N} =$ ________

This value is called the "correction factor" (C.F.).

74

Step E: Now compare the difference between the largest mean and the smallest mean with R_6 (since the range is over all 6 means).

This difference is (larger/smaller) than R_6, and therefore (is/is not) significant.

26a

Answer: $\frac{(\Sigma x_i)^2}{N} = \frac{(255)^2}{15}$

$= 4335$

27

Step D: Insert values into the formula.

$$SS_T = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{N}$$

$$SS_T = \# \_\_\_\_ - \# \_\_\_\_$$

$$= \# \_\_\_\_$$

74a

Answer: larger (33.0 and 6.642)
is

75

Now compare the difference between the largest and second smallest means with R_5 (now we're only talking about the range over 5 means).

This difference is (larger/smaller) than R_5, and therefore (is/is not) significant.

Go to:
Frame 75a
Page IV-30

Answer: 4555 - 4335

220

We now know SS_T and need to figure SS_B.

$$SS_B = \frac{(\Sigma x_1)^2}{n_1} + \frac{(\Sigma x_2)^2}{n_2} + \frac{(\Sigma x_3)^2}{n_3} - C.F.$$

x_1	x_2	x_3
12	13	20
13	14	21
14	15	22
15	16	23
16	17	24
70	75	110

Add each column and place in the formula.

SS_B = ____________________

What is C.F.? (Hint: See frames 26 and 26a)

____

$$SS_B = \frac{4900}{5} + \frac{5625}{5} + \frac{12100}{5} - \_\_\_\_$$

SS_B = # __________

28a

Answer: $SS_B = \frac{(70)^2}{5} + \frac{(75)^2}{5} + \frac{(110)^2}{5} - C.F.$

C.F. = 4335 (From formula for SS_T)

$SS_B = \frac{4900}{5} + \frac{5625}{5} + \frac{12100}{5} - 4335$

$= 980 + 1125 + 2420 - 4335$

$= 4525 - 4335$

$= 190$

29

Remember:

$SS_W = SS_T - SS_B$

SS_W = #_____ - #_____

= #_____

75a

Answer: larger (29.7 and 6.548)

is

76

Now compare the difference between the largest and third smallest means with R_4.

This difference is (larger/smaller) than R_4, and therefore (is/is not) significant.

29a

Answer: $SS_W = 220 - 190$

$= 30$

30

We now know SS_B, SS_W, and SS_T. In order to continue the computation of the analysis of variance we need to determine the between and within variance estimates. We do this by dividing each of the sums of squares by its degrees of freedom. The *df* are:

Between = $k - 1$ where k = number of samples

Within = $N - k$ where N = total number of values in all samples

Total = $N - 1$

We also at this time begin constructing an ANOVA summary table as shown below. Fill in the *df* in this table.

Source	SS	*df*
Between	190	#_____
Within	30	#_____
Total	220	#_____

76a

Answer: larger (23.4 and 6.420)
is

77

The next comparison is between the largest and fourth smallest with R_3.

(larger/smaller)

(is/is not) significant

30a

Answer:

Source	SS	df	
Between	190	2	(3 samples - 1)
Within	30	12	(15 total values - 3 samples)
Total	220	14	(15 total values - 1)

Note that both the SS and *df* columns are additive.

$$SS_B + SS_W = SS_T$$

$$df_B + df_W = df_T$$

77a

Answer: larger (12.1 and 6.236)

is

78

Now try the largest and fifth smallest with R_2.

(larger/smaller)

(is/is not)

31

Now that we know the sums of squares and their degrees of freedom, we can find the variance estimates (also known as the mean squares).

Source	SS	*df*	MS
Between	190	2	SS_B/df
Within	30	12	SS_W/df
Total	220	14	(We don't need to find MS for total)

Fill in MS:

Source	SS	*df*	MS
Between	190	2	# ____
Within	30	12	# ____
Total	220	14	

78a

Answer: larger (6.8 and 5.942)
is

79

We've run out of comparisons in that column so we move over to the difference between the second largest and the smallest and compare it to R_5. (Since we've eliminated the largest, we're only dealing with 5 means.)

(larger/smaller)
(is/is not)

Answer:

Source	SS	*df*	MS
Between	190	2	95
Within	30	12	2.5
Total	220	14	

32

Finally we find *F*. $F = \frac{MS_B}{MS_W}$

Source	SS	*df*	MS	F
Between	190	2	95	MS_B / MS_W
Within	30	12	2.5	
Total	220	14		

Fill in *F*:

Source	SS	*df*	MS	*F*
Between	190	2	95	# ______
Within	30	12	2.5	
Total	220	14		

79a

Answer: larger (26.2 and 6.548)
is

80

Continue making these comparisons until you've compared all differences with the appropriate R.

.
.
.
.
.
.

Did you find any differences that were not significant? ______________

Which ones? + ________________

32a

Answer: $F = 38.0$

33

To test $F = 38.0$ for significance at the 5% level, we need to find the critical value. Go to Table G and find the critical value for F with $k - 1$ and $N - k$ degrees of freedom. (Remember the first df is across the top and the second is down the side.)

$$F_{2,12} = \# \_\_\_\_\_\_$$

80a

Answer: yes

the differences between $\bar{x}_4$ and $\bar{x}_5$

and $\bar{x}_1$ and $\bar{x}_2$

81

Typically, non-significant differences are underscored in the table as shown below:

	$\bar{x}_1$ 36.4	$\bar{x}_2$ 39.7	$\bar{x}_3$ 46.0	$\bar{x}_4$ 57.3	$\bar{x}_5$ 62.6	$\bar{x}_6$ 69.4	Shortest Significant Ranges
$\bar{x}_1$ 36.4		3.3	9.6	20.9	26.2	33.0	$R_2 = 5.942$
$\bar{x}_2$ 39.7			6.3	17.6	22.9	29.7	$R_3 = 6.236$
$\bar{x}_3$ 46.0				11.3	16.6	23.4	$R_4 = 6.420$
$\bar{x}_4$ 57.3					5.3	12.1	$R_5 = 6.548$
$\bar{x}_5$ 62.6						6.8	$R_6 = 6.642$
$\bar{x}_6$ 69.4							

Any two means not underscored by the same line are significantly different. (e.g., since x_5 and x_6 are not underscored by the same line, they are significantly different; and since x_4 and x_5 are underscored by the same line they are not significantly different.)

3a

Answer: 3.89

If you got this wrong, you may have used the 1% table. Check again.

34

Since our obtained F (38.0) is larger than the critical value (3.89), we (accept/reject) the H_o: $\mu_1 = \mu_2 = \mu_3$.

82

Before you try a couple on your own, answer this question:

How can you find n given an ANOVA summary table? (Hint: see df column.) Remember that equal n's are necessary for the Duncan Test.

† ______________________________

Answer: reject

35

One of the more frequently asked questions about ANOVA is why can an F test be used to test a hypothesis about means when an F test is a ratio of variances? Let's see if we can come up with an answer for that.

82a

Answer: Since $df_{TOTAL} = N-1$, we can add one to this value and divide by the number of samples, k. For example:

	df	
Between	3 = $(k-1)$.	Therefore, $k = 4$.
Within	20	
Total	23	

$$\frac{23 + 1}{4} = \frac{24}{4} = 6$$

$$n = 6$$

83

Given the following, determine any means that differ significantly at the 5% level.

$\bar{x}_1 = 13.6$ $\bar{x}_2 = 14.0$ $\bar{x}_3 = 14.4$ $\bar{x}_4 = 33.2$ $\bar{x}_5 = 12.9$

Source	SS	df	MS	F
Between	946.0	4	236.50	9.46
Within	500.0	20	25.00	
Total	1446.0	24		

Step A: Rank order the means.

First of all, let's look at an ANOVA summary table.

Source	SS	*df*	MS	*F*
Between				
Within				
Total				

The figures we obtain in the mean squares column are actually variance estimates. To test the significance of the ratio of two variance estimates, we use an *F* test. If the between and within variance estimates are equal, their ratio is 1.00. If the null hypothesis ($H_o: \mu_1 = \mu_2 = \mu_3 = \ldots\mu_n$) is false, the ratio of these estimates is > 1.00. The determination of how much greater than 1.00 is significant depends on the α level selected and the degrees of freedom associated with the two estimates.

83a

Answer:

12.9, 13.6, 14.0, 14.4, 33.2

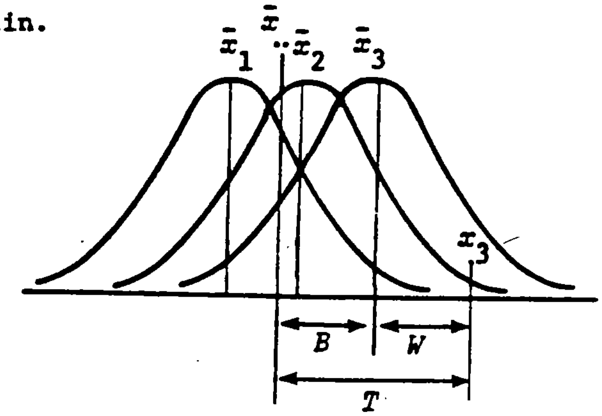
84

Step B: Find $s_{\bar{x}}$

$$s_{\bar{x}} = \frac{s}{\sqrt{n}}$$

$s_{\bar{x}} =$ # ________

Let's try something that may help explain the previous frame. Assume we have three samples again.



Just by eyeing the three distributions, what does the ratio of $\frac{B}{W}$ look like?

A. $\frac{2}{1}$

B. $\frac{3}{1}$

C. $\frac{1}{1}$

84a

Answer:

$$s_{\bar{x}} = \frac{s}{\sqrt{n}}$$

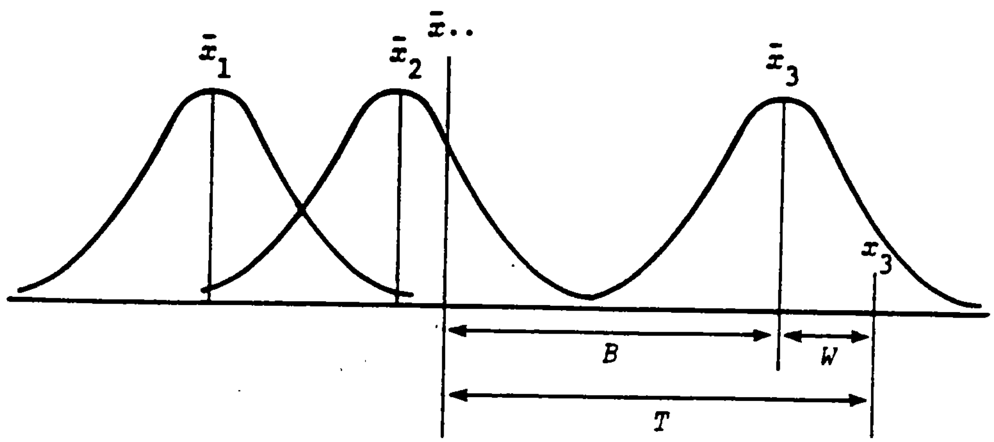
$$= \frac{5}{\sqrt{5}}$$

$$= 2.24$$

Answer: C. $\frac{1}{1}$

38

Let's change our sample distributions a bit.



Assume the three sample distributions remained the same, except the values in sample 3 have been increased by a constant amount. (Variances within each sample have been unchanged.) T and B have (increased, decreased, remained the same). W has (increased, decreased, remained the same).

85

Step C: Determine the shortest significant ranges.

R_2 = # ______

R_3 = # ______

R_4 = # ______

R_5 = # ______

Answer: *T* and *B* have increased

W has remained the same

39

In any ratio, $\frac{x}{y}$, if x is increased, the result is also increased.

Since the *F* ratio in ANOVA is based on SS_B/SS_W (before division by *df*), it stands to reason that if SS_B is increased and SS_W remains the same, the *F* obtained would also be increased. In our second example, it is obvious that $\bar{x}_3$ differs considerably from $\bar{x}_1$ and $\bar{x}_2$, and so we would expect a (high/low) *F* value.

85a

Answer:

$R_2 = 2.950\ (2.24) = 6.608$

$R_3 = 3.097\ (2.24) = 6.937$

$R_4 = 3.190\ (2.24) = 7.146$

$R_5 = 3.255\ (2.24) = 7.291$

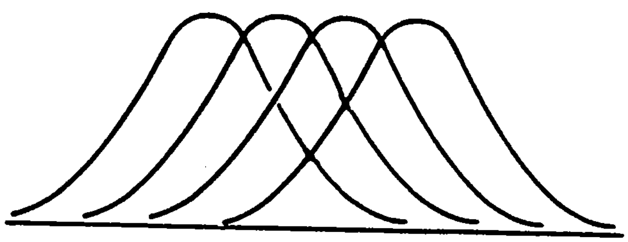
Go to:

Frame 86

Page IV-43

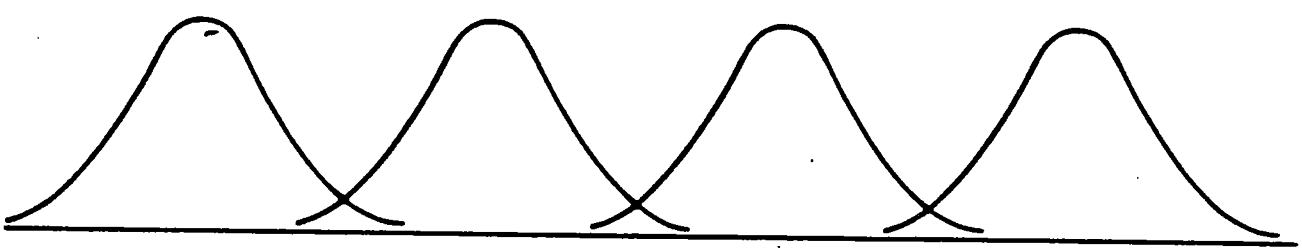
Answer: high (perhaps significant--without data we can't tell)

If our data have these distributions



we would expect a relatively (high/low) *F* value.

If our data look like this,



we would expect a relatively (high/low) *F* value.

40a

Answer: low

high

41

So you see if the sample means are considerably different, this difference would be found by an ANOVA due to the way in which the total variability is partitioned into between sample and within sample sources.

86

Step D: Construct the table.

	$\bar{x}$	$\bar{x}$	$\bar{x}$	$\bar{x}$	$\bar{x}$	Shortest Significant Ranges
$\bar{x}$						R_2 =
$\bar{x}$						R_3 =
$\bar{x}$						R_4 =
$\bar{x}$						R_5 =
$\bar{x}$						

42

Try this one on your own:

	x_1	x_1^2	x_2	x_2^2	x_3	x_3^2	x_4	x_4^2
	22		24		16		22	
	26		19		27		28	
	23		23		28		17	
	22		23		17		25	
Σ								

Sum each column

86a

Answer:

		$\bar{x}_5$ 12.9	$\bar{x}_1$ 13.6	$\bar{x}_2$ 14.0	$\bar{x}_3$ 14.4	$\bar{x}_4$ 33.2	Shortest Significant Ranges
$\bar{x}_5$	12.9		.7	1.1	1.5	20.3	R_2 = 6.608
$\bar{x}_1$	13.6			.4	.8	19.6	R_3 = 6.937
$\bar{x}_2$	14.0				.4	19.2	R_4 = 7.146
$\bar{x}_3$	14.4					18.8	R_5 = 7.291
$\bar{x}_4$	33.2						

87

Step E: Compare the differences between means with the shortest significant ranges.

Underscore non-significant differences.

Answer:

x_1	x_1^2	x_2	x_2^2	x_3	x_3^2	x_4	x_4^2
22	484	24	576	16	256	22	484
26	676	19	361	27	729	28	784
23	529	23	529	28	784	17	289
22	484	23	529	17	289	25	625
93	2173	89	1995	88	2058	92	2182

43

Step A: Find SS_T $\quad SS_T = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{N}$

Go to:

Frame 43a

Page IV-47

87a

Answer:

		$\bar{x}_5$ 12.9	$\bar{x}_1$ 13.6	$\bar{x}_2$ 14.0	$\bar{x}_3$ 14.4	$\bar{x}_4$ 33.2	Shortest Significant Ranges
$\bar{x}_5$	12.9		.7	1.1	1.5	20.3	R_2 = 6.608
$\bar{x}_1$	13.6			.4	.8	19.6	R_3 = 6.937
$\bar{x}_2$	14.0				.4	19.2	R_4 = 7.146
$\bar{x}_3$	14.4					18.8	R_5 = 7.291
$\bar{x}_4$	33.2						

$\bar{x}_4$ differs significantly from all the others (not too surprising, was it?). $\bar{x}_5$, $\bar{x}_1$, $\bar{x}_2$, $\bar{x}_3$, do not differ from each other.

Let's get back to the analysis of variance.

The ANOVA you have just studied is called a one-way ANOVA. There are, however, many other designs for analysis of variance suited for special circumstances. These include:

- Two-factor ANOVA
  - fixed effects
  - random effects
  - mixed effects
- Multi-factor ANOVA
  - fixed effects
  - random effects
  - mixed effects
- Two-factor ANOVA with repeated measures on one factor
- Multiple-factor ANOVA with repeated measures on one or more factors

and more.

Again, we are not going to cover these in this course. They are more suited for an advanced course. Information on them is available in the following:

1. Guenther, William C. Analysis of variance. Englewood Cliffs, N. J.: Prentice-Hall, 1964.

2. Li, C. C. Introduction to experimental statistics. New York: McGraw-Hill, 1964.

3. Winer, B. J. Statistical principles in experimental design (2nd Edition). New York: McGraw-Hill, 1971.

4. Kirk, Roger E. Experimental design: Procedures for the behavioral sciences. Belmont, Cal.: Brooks/Cole, 1968.

5. Scheffe, Henry. The analysis of variance. New York: Wiley, 1959.

43a

Step A: $SS_T = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{N}$ $\qquad \Sigma x_i^2 = 2173 + 1995 + 2058 + 2182$

$\Sigma x_i = 93 + 89 + 88 + 92$

$= 8408 - \frac{(362)^2}{16}$

$= 8408 - 8190.25$ $\qquad$ C.F. $= 8190.25$

$= 217.75$

44

Step B: Find SS_B $\qquad SS_B = \frac{(\Sigma x_1)^2}{n_1} + \frac{(\Sigma x_2)^2}{n_2} + \frac{(\Sigma x_3)^2}{n_3} + \frac{(\Sigma x_4)^2}{n_4} - \text{C.F.}$

89

Let's try something now. What do you suppose would happen if we tried ANOVA on two sample means (we'd normally use a *t* test for this)? Let's compute the two simultaneously.

Step B: $$SS_B = \frac{(\Sigma x_1)^2}{n_1} + \frac{(\Sigma x_2)^2}{n_2} + \frac{(\Sigma x_3)^2}{n_3} + \frac{(\Sigma x_4)^2}{n_4} - C.F.$$

$$= \frac{(93)^2}{4} + \frac{(89)^2}{4} + \frac{(88)^2}{4} + \frac{(92)^2}{4} - 819\iota$$

$$= 2162.25 + 1980.25 + 1936.0 + 2116.0 - 8190.25$$

$$= 8194.5 - 8190.25$$

$$= 4.25$$

Step C: Find SS_W $\quad SS_T - SS_B = SS_W$

Compute t and ANOVA on the following independent sample data.

x_1	x_2
20	27
21	29
25	31
19	30
15	33

t obtained = # ______

F obtained = # ______

$$t_{N-2} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\left[\Sigma x_1^2 - \frac{(\Sigma x_1)^2}{n_1}\right] + \left[\Sigma x_2^2 - \frac{(\Sigma x_2)^2}{n_2}\right]}{n_1 + n_2 - 2}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

ANOVA

$$SS_T = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{N}$$

$$SS_B = \frac{(\Sigma x_1)^2}{n_1} + \frac{(\Sigma x_2)^2}{n_2} + \ldots + \frac{(\Sigma x_n)^2}{n_n} - C.F.$$

$$SS_W = SS_T - SS_B$$

45a

Step C: $SS_W = SS_T - SS_B$

$= 217.75 - 4.25$

$= 213.5$

46

Step D: Enter into table

Source	SS	*df*	MS	*F*
Between				
Within				
Total				

90a

Answer: $t = -5.271$

$F = 27.78$

91

Square t obtained

$t^2 =$ ____________

46a

Step D:

Source	SS	*df*	MS	*F*
Between	4.25	3	1.42	.08
Within	213.50	12	17.79	
Total	217.75	15		

47

Step E: Test obtained *F* against critical value.

Go to:

Frame 47a

Page IV-53

91a

Answer: $t^2 = 27.783$

That's right, in a two-sample test $t^2 = F$.

92

Let's try a couple for practice. Compute ANOVA on the following problems, and, if a significant *F* is found, perform Duncan's Test as well.

A study is being performed to see if the location around a large metropolitan area has any effect on NO_2 concentrations. In order to do this the following data (in $\mu g/m^3$) were collected:

N.Y.C.	Jersey City, N.J.	Bridgeport, Conn.
248	245	316
130	170	163
100	169	116
196	190	191
106	197	121
147	219	171

Test this data to determine if the mean NO_2 concentrations are different at the 5% level of significance.

Answer:

STEP 1: Define the problem

H_o: $\mu_1 = \mu_2 = \mu_3$

H_1: not all μ's are equal

STEP 2: Select technique and set α level

One-Way ANOVA $\alpha = .05$

STEP 3: Organize and arrange data

x_1	x_1^2	x_2	x_2^2	x_3	x_3^2
248	61504	245	60025	316	99856
130	16900	170	28900	163	26569
100	10000	169	28561	116	13456
196	38416	190	36100	191	36481
106	11236	197	38809	121	14641
147	21609	219	47961	171	29241
927	159665	1190	240356	1078	220244

STEP 4: Compute the statistic

$$SS_T = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{N}$$

$$= 159665 + 240356 + 220244 - \frac{(927 + 1190 + 1078)^2}{18}$$

$$= 620265 - 567112.50$$

$$= 53152.50$$

$$SS_B = \frac{(\Sigma x_1)^2}{n_1} + \frac{(\Sigma x_2)^2}{n_2} + \ldots + \frac{(\Sigma x_n)^2}{n_n} - \text{C.F.}$$

$$= \frac{(927)^2}{6} + \frac{(1190)^2}{6} + \frac{(1078)^2}{6} - \frac{(927 + 1190 + 1078)^2}{18}$$

$$= 143221.5 + 236016.67 + 193680.67 - 567112.5$$

$$= 5806.33$$

$$SS_W = SS_T - SS_B$$

$$= 53152.5 - 5806.33$$

$$= 47346.17$$

92a continued page IV-52

Source	SS	*df*	MS	*F*
Between	5806.33	2	2903.16	.92
Within	47346.17	15	3156.41	
Total	53152.50	17		

$df_B = k - 1 = 3 - 1 = 2$

$df_W = N - k = 18 - 3 = 15$

$df_T = N - 1 = 18 - 1 = 17$

STEP 5: Determine critical value

Critical Value = 3.68

STEP 6: Make the appropriate decision

Accept H_o since the obtained value is less than the tabled value.

47a

Step E: Critical value =

$$F_{3,12} = 3.49$$

Since obtained F is less than 3.49 we accept the H_o:

$$\mu_1 = \mu_2 = \mu_3 = \mu_4$$

Go to:

Frame 48

Page IV-57

93

Four different monitoring devices were placed in the same spot and recorded CO readings (ppm) at the same time. The following are the collected data:

Device number

1	2	3	4
5	7	19	7
9	11	17	13
13	11	15	11
9	8	18	7
4	5	13	3

Can we conclude from these data that the mean readings by the four devices differ at the 5% level of significance?

Answer:

STEP 1: Define the problem

H_o: $\mu_1 = \mu_2 = \mu_3 = \mu_4$

H_1: not all μ's are equal

STEP 2: Select technique and set α level

One-Way ANOVA $\alpha = .05$

STEP 3: Organize and arrange data

x_1	x_1^2	x_2	x_2^2	x_3	x_3^2	x_4	x_4^2
5	25	7	49	19	361	7	49
9	81	11	121	17	289	13	169
13	169	11	121	15	225	11	121
9	81	8	64	18	324	7	49
4	16	5	25	13	169	3	9
40	372	42	380	82	1368	41	397

STEP 4: Compute the statistic

$$SS_T = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{N}$$

$$= 2517 - \frac{42025}{20}$$

$$= 2517 - 2101.25$$

$$= 415.75$$

$$SS_B = \frac{(\Sigma x_1)^2}{n_1} + \frac{(\Sigma x_2)^2}{n_2} + \ldots + \frac{(\Sigma x_n)^2}{n_n} - \text{C.F.}$$

$$= \frac{(40)^2}{5} + \frac{(42)^2}{5} + \frac{(82)^2}{5} + \frac{(41)^2}{5} - 2101.25$$

$$= 320 + 352.8 + 1344.8 + 336.2 - 2101.25$$

$$= 252.55$$

$$SS_W = SS_T - SS_B$$

$$= 415.75 - 252.55$$

$$= 163.20$$

93a continued page IV-55

Source	SS	df	MS	F
Between	252.55	3	84.183	8.253
Within	163.20	16	10.200	
Total	415.75	19		

STEP 5: Determine critical value

Critical Value = 3.24

STEP 6: Make the appropriate decision

Reject H_o since the obtained value is greater than the critical value.

Duncan's Test

Step A: Rank order the means.

8.0, 8.2, 8.4, 16.4

Step B: Find $s_{\bar{x}}$.

$$s_{\bar{x}} = \frac{s}{\sqrt{n}}$$

$$= \frac{\sqrt{10.2}}{\sqrt{5}}$$

$$= 1.428$$

Step C: Determine shortest significant ranges.

$R_2 = 2.998 \ (1.428) = 4.281$

$R_3 = 3.144 \ (1.428) = 4.489$

$R_4 = 3.235 \ (1.428) = 4.619$

Step D: Construct Table.

		$\bar{x}_1$ 8.0	$\bar{x}_4$ 8.2	$\bar{x}_2$ 8.4	$\bar{x}_3$ 16.4	Shortest Significant Ranges
$\bar{x}_1$	8.0		.2	.4	8.4	$R_2 = 4.281$
$\bar{x}_4$	8.2			.2	8.2	$R_3 = 4.489$
$\bar{x}_2$	8.4				8.0	$R_4 = 4.619$
$\bar{x}_3$	16.4					

93a continued page IV-56

Step E: The means underscored by the same line are not significantly different.

Therefore, the mean of device 3 is significantly different from all the others. (Remember we rank ordered the means, and the largest belonged to device 3.)

Let's back up a minute. Suppose we conduct ANOVA on four samples and obtain a significant *F*. We then reject H_o: $\mu_1 = \mu_2 = \mu_3 = \mu_4$. All this means is that all 4 means are not equal, which leaves the possibility that only one is not equal to the other three, or two may not equal the other two, etc. In other words, rejection of H_o: $\mu_1 = \mu_2 = \mu_3 = \mu_4$ could be due to any of the following conditions:

$$\mu_1 \neq \mu_2 = \mu_3 = \mu_4$$
$$\mu_1 = \mu_2 \neq \mu_3 = \mu_4$$
$$\mu_1 = \mu_2 = \mu_3 \neq \mu_4$$
$$\mu_1 \neq \mu_2 \neq \mu_3 = \mu_4$$
$$\mu_1 \neq \mu_2 = \mu_3 \neq \mu_4$$
$$\mu_1 \neq \mu_2 \neq \mu_3 \neq \mu_4$$
$$\mu_1 = \mu_2 \neq \mu_3 \neq \mu_4$$

and others.

Wouldn't it be nice to be able to compare all possible combinations of the means to determine which difference(s) caused the rejection of H_o?

The analysis of variance is a very useful technique, since it allows the statistician to compare means of two or more samples. What's more, there are many different kinds of ANOVA to handle almost any experimental design. Several of these were mentioned, but only one--the one-way ANOVA (also called the "single factor," "simple," or "single-classification ANOVA") was presented in any detail.

The one-way ANOVA tests the null hypothesis

$$H_o: \mu_1 = \mu_2 = \mu_3 = \ldots . \mu_n$$

against the alternative hypothesis

$$H_1: \text{not all } \mu\text{'s are equal}$$

Go to:

Frame 95

Page IV-59

48a

The answer to this one is, of course, yes.

49

It just so happens there are ways to do just that. They include Duncan's New Multiple Range Test, the Tukey and Scheffé tests of multiple comparisons, and others. In this course, we will only present Duncan's Test in detail. Additional information on the others, however, can be found in any of the following references:

1. Guenther, William C. Analysis of variance. Englewood Cliffs, N.J.: Prentice-Hall, 1964.
2. Li, C. C. Introduction to experimental statistics. New York: McGraw-Hill, 1964.
3. Winer, B. J. Statistical principles in experimental design (2nd Edition). New York: McGraw-Hill, 1971.
4. Kirk, Roger E. Experimental design: Procedures for the behavioral sciences. Belmont, Cal.: Brooks/Cole, 1968.
5. Scheffé, Henry, The Analysis of variance. New York: Wiley, 1959

Go to:

Frame 50

Page IV-1

To perform an analysis, the following sums of squares need to be calculated:

$$SS_{TOTAL} = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{N}$$

where x_i = each individual score in all samples

N = total number of all sample values

$$SS_{BETWEEN} = \frac{(\Sigma x_1)^2}{n_1} + \frac{(\Sigma x_2)^2}{n_2} + \ldots + \frac{(\Sigma x_n)^2}{n_n} - C.F.$$

where x_1 = each individual score in sample 1

x_2 = each individual score in sample 2

C.F. = correction factor = $\frac{(\Sigma x_i)^2}{N}$ in the above formula for SS_{TOTAL}

$$SS_{WITHIN} = SS_{TOTAL} - SS_{BETWEEN}$$

These values are then put into an ANOVA summary table.

Source	SS	df	MS	F
Between		$k - 1$	SS_B/df_B	MS_B/MS_W
Within		$N - k$	SS_W/df_W	
Total		$N - 1$		

Where k = number of samples

The F obtained by the ratio $\frac{MS_B}{MS_W}$ is then compared with the tabled critical value F with $k - 1$ and $N - k$ degrees of freedom ($F_{k-1,N-k}$). If the obtained F exceeds the critical value then the H_o: $\mu_1 = \mu_2 = \mu_3 = \ldots \mu_n$ is rejected. At this point, to determine which mean(s) are responsible for the rejection, a separate test needs to be conducted.

This module discussed Duncan's Test of Multiple Comparisons.

Duncan's Test consists of comparing the differences between the sample means with the shortest significant ranges and is performed in the following steps:

Step A: Rank order the sample means.

Step B: Find the standard error of the mean.

$$s_{\bar{x}} = \frac{s}{\sqrt{n}} \quad \text{where } s = \sqrt{MS_{within}} \text{ from the ANOVA summary table}$$

Step C: Determine the shortest significant ranges. The shortest significant ranges are equal to the product of the standard error of the mean times the significant studentized ranges found in Table H of the Guide.

Step D: Construct a table of differences of means.

Step E: Compare the differences between means with the appropriate shortest significant range to test for significance.

Go to Module V

	$\bar{x}_1$ 36.4	$\bar{x}_2$ 39.7	$\bar{x}_3$ 46.0	$\bar{x}_4$ 57.3	$\bar{x}_5$ 62.6	$\bar{x}_6$ 69.4	Shortest Significant Ranges
$\bar{x}_1$ 36.4		3.3	9.6	20.9	26.2	33.0	R_2 = 5.942
$\bar{x}_2$ 39.7			6.3	17.6	22.9	29.7	R_3 = 6.236
$\bar{x}_3$ 46.0				11.3	16.6	23.4	R_4 = 6.420
$\bar{x}_4$ 57.3					5.3	12.1	R_5 = 6.548
$\bar{x}_5$ 62.6						6.8	R_6 = 6.642
$\bar{x}_6$ 69.4							

Leave this frame folded out for use with frames 74-80.