7 STATISTICAL BASIS FOR MARSAME SURVEYS

The statistically rigorous quantitative application of measurement quality objectives (MQOs) plays a central role in the MARSAME process. MQOs did not appear explicitly in *Multi-Agency Radiation Survey and Site Investigation Manual* (MARSSIM 2002), but were subsequently developed for radioanalytical chemistry measurements as part of the *Multi-Agency Radiological Laboratory Analytical Protocols* (MARLAP) manual. However, these concepts apply equally well to field measurements of radiation and radioactivity. The MARSAME process incorporates these ideas and extends them to these measurements.

A major development since the publication of MARSSIM was the publication of the *Guide to the Expression of Uncertainty in Measurement*, or "GUM" (ISO 1995). The procedures described in this document have become a de facto standard for estimating the uncertainty associated with measurements of any type. The GUM methodology is essential for the assessment of measurement uncertainty, but was not previously treated in MARSSIM.

Data quality objectives (DQO) form the backbone of the MARSAME process, and are discussed in detail in Chapters 2 and 3. A number of terms with specific statistical meanings are used in this and subsequent sections. The concept of measurement quality objectives (MQOs) and in particular the required measurement method uncertainty was introduced in Section 3.8. These ideas are discussed in greater detail in MARLAP Chapter 3 and Appendix C. While MARLAP is focused on radioanalytical procedures, these concepts are applicable on a much broader scale and are used in MARSAME in Sections 5.5 through 5.8 to guide the selection of measurement methods for disposition surveys for materials and equipment.

In Section 7.1 the general concepts of statistical survey design and hypothesis testing are discussed, with more detail in Section 7.2. In Sections 7.3, 7.4, 7.5, and 7.6, calculation of measurement quality objectives (particularly the required method uncertainty), measurement uncertainty, minimum detectable concentrations (MDCs) and minimum quantifiable concentrations (MQCs), respectively, are introduced. Further details and examples of these topics for the interested reader are then given in Sections 7.7, 7.8, 7.9, and 7.10. This advanced material is optional on initial reading, and may be referred to later as needed. Section 7.11 shows a detailed calculation of a scan MDC, which is used in Chapter 8. This process was described and used in MARSSIM, but a systematic example was constructed for M&E. These calculations are detailed, and are also optional on first reading.

In developing the results in this chapter, a number of new and sometimes only subtly different definitions and symbols are used. For the convenience of the reader, many of these are summarized in the tables below. Table 7.1 provides a summary of notation used for DQOs and MQOs, used primarily in Sections 7.1 and 7.2. Table 7.2 contains notation used for setting MQOs for required method uncertainties (Sections 7.3 and 7.7) and in uncertainty calculations (Sections 7.4 and 7.8). MDC calculations (Sections 7.5 and 7.9) and MQC calculations (Sections 7.6 and 7.10) use the notation added in Table 7.3 and Table 7.4, respectively. Symbols may not have an entry for both formula or reference and type.

Table 7.1 Notation for DQOs and MQOs

Symbol	Definition	Formula or reference	Type
α	Probability of a Type I decision error		Chosen during DQO process
β	The probability of a Type II decision error		Chosen during DQO process
Δ	Width of the gray region	(UBGR-LBGR)	Chosen during DQO process
φ_{MR}	Required relative method uncertainty above the UBGR	u_{MR}/UBGR	Chosen during DQO process
S_C	The critical value of the net instrument signal (e.g., net count)	Calculation of S_C requires the choice of a significance level for the test. The significance level is a specified upper bound for the probability, α, of a Type I error. The significance level is usually chosen to be 0.05.	If a measured value exceeds the critical value, a decision is made that radiation or radioactivity has been detected
$\hat{S}$	net signal		Experimental
σ	The total standard deviation of the data	$(\sigma_S^2 + \sigma_M^2)^{½}$	Theoretical population parameter
σ_N	The standard deviation of the mean of N independent measurements	$\sigma_N = \sigma/\sqrt{N}$	
σ_S	Standard deviation due to sampling		Theoretical population parameter
σ_M	Standard deviation of the measurement method		Theoretical population parameter
σ_{MR}	Required method standard deviation at and below the UBGR	Upper bound to the value of σ_M	Theoretical population parameter
u_{MR}	Required method uncertainty at and below the UBGR	Upper bound to the value of u_M	Chosen during DQO process
$u_c^2(y)$	Combined variance of y	Uncertainty propagation	Calculated
$u_c(y)$	Combined standard uncertainty of y	Uncertainty propagation	Calculated
$z_{1-\alpha}$ $(z_{1-\beta})$	$1-\alpha$ (or $1-\beta$) quantile of a standard normal distribution function	Table of standard normal distribution	Theoretical

Table 7.2 Notation for Uncertainty Calculations

Symbol	Definition	Formula or reference	Type
a	Half-width of a bounded probability distribution	Type B evaluation of uncertainty	Estimated
c_i	Sensitivity coefficient	$\partial f / \partial x_i$, the partial derivative of f with respect to x_i	Evaluated at the measured values $x_1,x_2,\ldots,x_N$
$f(x_1, x_2,\ldots,x_N)$	The calculated value of the output quantity from measurable input quantities for a particular measurement	$y = f(x_1, x_2,\ldots,x_N)$	Experimental
$f(X_1, X_2,\ldots,X_N)$	Model equation expressing the mathematical relationship, between the measurand, Y and the input quantities X_i	$Y=f(X_1, X_2,\ldots,X_N)$	Theoretical
k	Coverage factor for expanded uncertainty	Numerical factor used as a multiplier of the combined standard uncertainty in order to obtain an expanded uncertainty	Chosen during DQO process
p	Coverage probability for expanded uncertainty	Probability that the interval surrounding the result of a measurement determined by the expanded uncertainty will contain the value of the measurand	Chosen during DQO process
$r(x_i,x_j)$	Correlation coefficient for two input estimates, x_i and x_j	$u(x_i,x_j) / (u(x_i)\, u(x_j))$	Experimental
$s(x_i)$	Sample standard deviation of the input estimate x_i	$s(x_i)=\sqrt{\frac{1}{(n-1)}\sum_{k=1}^{n}(x_{i,k}-\overline{x_i})^2}$	Experimental
$u(x_i)$	Type B standard uncertainty of the input estimate x_i		Estimated
$u_i(y)$	Component of the combined standard uncertainty $u_c(y)$ generated by the standard uncertainty of the input estimate x_i, $u(x_i)$	$u_i(y) = c_i\, u(x_i)$	Estimated
$u_c(y)$	Combined standard uncertainty of y	Uncertainty propagation	Calculated
$u_c^2(y)$	Combined variance of y	Uncertainty propagation	Calculated
U	Expanded uncertainty	"Defining an interval about the result of a measurement that may be expected to encompass a large fraction of values that could reasonably be attributed to the measurand" (GUM)	Calculated
$u(x_i,x_j)$	Covariance of two input estimates, x_i and x_j		Experimental

Table 7.2 Notation for Uncertainty Calculations (Continued)

Symbol	Definition	Formula or reference	Type
$u_c(y)/y$	Relative combined standard uncertainty of the output quantity for a particular measurement		Experimental
$u(x_i)/x_i$	Relative standard uncertainty of a nonzero input estimate x_i for a particular measurement		Experimental
$w_1,w_2,\ldots,w_N$	Input quantities appearing in the numerator of $y = f(x_1, x_2,\ldots,x_N)$	See "$z_1,z_2,\ldots,z_N$" below	
$X_1, X_2,\ldots,X_N$	Measurable input quantities		Theoretical
$x_1, x_2,\ldots, x_N$	Estimates of the measurable input quantities for a particular measurement		Experimental
Y	The output quantity or measurand		Theoretical
y	Estimate of the output quantity for a particular measurement		Experimental
$z_1,z_2,\ldots,z_N$	Input quantities appearing in the denominator of $y = f(x_1, x_2,\ldots,x_N)$	$N = n+m$	Experimental

Table 7.3 Notation for MDC Calculations

Symbol	Definition	Formula or reference	Type
N_B	Background count		Experimental
N_S	Gross sample count		Experimental
t_S	Count time for the test source or sample		Experimental
t_B	Count time for the background		Experimental
R_B	Mean count rate of the blank	$R_B = \frac{N_B}{t_B}$	
d	Parameter in the Stapleton equation for the critical value of the net instrument signal	Usually has the value 0.4	
ε	Efficiency	Calibration	Experimental or Theoretical
F	Calibration function	$X = F(Y)$	
F^{-1}	Evaluation function	$Y = F^{-1}(X)$, closely related to the mathematical model $Y=f(X_1, X_2,\ldots,X_N)$	

Table 7.3 Notation for MDC Calculations (Continued)

Symbol	Definition	Formula or reference	Type
S_C	Critical value of the net instrument signal	Net instrument signal is calculated from the gross signal by subtracting the estimated background and any interferences	
S_D	Minimum detectable value of the net instrument signal	Net instrument signal that gives a specified probability, 1−β, of yielding an observed signal greater than its critical value S_C	
X	Observable response variable, measurable signal		Experimental
x_C	The critical value of the response variable	Calculation of y_C requires the choice of a significance level for the test. The significance level is a specified upper bound for the probability, α, of a Type I error. The significance level is usually chosen to be 0.05.	If a measured value exceeds the critical value, a decision is made that radiation or radioactivity has been detected
Y	State variable, measurand	Uncertainty propagation	
y_C	Critical value of the concentration	$y_C = F^{-1}(x_C)$	
y_D	Minimum detectable concentration (MDC)	$y_D = \frac{S_D}{\varepsilon}$	
Δ_B	Relative systematic error in the background determination		Experimental
Δ_A	Relative systematic error in the sensitivity		Experimental

Table 7.4 Notation for MQC Calculations

Symbol	Definition	Formula or reference	Type
k_Q	Multiple of the standard deviation defining y_Q, usually chosen to be 10	$k_Q = \frac{\sqrt{\sigma^2(y \mid Y = y_Q)}}{y_Q}$	Chosen during DQO process
$\sigma^2(y \mid Y = y_Q)$	The variance of y given the true concentration Y equals y_Q		Theoretical
y_Q	Minimum quantifiable concentration (MQC)	The concentration at which the measurement process gives results with a specified relative standard deviation $1/k_Q$, where k_Q is usually chosen to be 10	Theoretical
R_I	Mean interference count rate		Experimental

Table 7.4 Notation for MQC Calculations (Continued)

Symbol	Definition	Formula or reference	Type
$\sigma(\widehat{R}_I)$	Standard deviation of the measured interference count rate		Experimental
$\phi^2_{\hat{\varepsilon}}$	Relative variance of the measured efficiency, $\hat{\varepsilon}$		Experimental

7.1 Overview of Statistical Survey Design and Hypothesis Testing

Designing a MARSAME survey involves the following key statistical parameters:

(1) The uncertainty in the measurement method. The measurement method uncertainty can be affected by changes to the measurement method, such as changing counting times, or performing repeated measurements. Generally, the measurement method uncertainty is characterized by its standard deviation, σ_M. This value may be a constant, meaning that all measurements will have the same standard deviation. Alternatively, this value may vary with the level of radionuclide concentration or radioactivity, such that the standard deviation increases with increasing radionuclide concentration or radioactivity.

(2) The uncertainty in the distribution of radionuclide concentrations or radioactivity in the population of materials and equipment (M&E) to be measured. This variation of radionuclide concentrations or radioactivity in space and time can be characterized by the sampling standard deviation, σ_S.

(3) The number of samples, *N*, from the population of radionuclide concentrations or radioactivity that comprises the survey unit.

(4) The null (H_0) and alternative (H_1) hypotheses to be examined. The symbol Δ represents the detectable difference between the null hypothesis concentration value (the action level, or AL), and the alternative hypothesis concentration value (the discrimination limit, or DL). The range of concentrations between the AL and the DL is referred to as the gray region.

(5) The values of α and β that quantify acceptable limits for Type I and Type II decision errors, respectively. A Type I decision error occurs when the null hypothesis is rejected when it is actually true. A Type II decision error occurs when the null hypothesis is not rejected but should have been rejected. The value of $1-\beta$ is termed the power, or the ability of the statistical test to reject the null hypothesis, when appropriate. For a specific survey design, the power ($1-\beta$) of the survey can be compared at different values of α, since the power is the probability of rejecting the null hypothesis at a given value of α.

Note: Designing a survey involves collecting a number of measurements, N, that will yield the desired α and power ($1-\beta$), given a detectable difference Δ, the σ_M for the measurement method selected and the σ_S for the distribution of radionuclide concentrations or radioactivity in the population of materials and equipment (M&E) to be measured. The relationships between these parameters are complex and interrelated. The choice or determination of one parameter affects the choice or determination of the other parameters.

When a single measurement is taken, the variance of that measurement will equal:

$$\sigma^2 = \sigma^2_M + \sigma^2_S \quad (7\text{-}1)$$

In some cases, the distribution of radionuclide concentrations or radioactivity in the population of M&E to be measured and thus σ_S may not be important to a MARSAME survey, e.g., in cases where there is no sampling variability. It then becomes important how the measurement method uncertainty changes when repeated measurements of the same sampling unit are taken. It may be reasonable to assume that the mean of *N* independent measurements of the same sampling unit will have a standard deviation:

$$\sigma_N = \sigma_M/\sqrt{N} \quad (7\text{-}2)$$

When variability in the distribution of radionuclide concentrations or radioactivity in the population of M&E to be measured occurs over time and space, then σ_S is not equal to zero, and must be included in the MARSAME survey design. The variance of the mean of a random sample of *N* measurements will fall in a range between

$$\sigma_N^2 = [\sigma^2_M + \sigma^2_S]/N \quad (7\text{-}3)$$

and

$$\sigma_N^2 = \sigma^2_M + \sigma^2_S/N \quad (7\text{-}4)$$

Equation 7.3 corresponds to measurement method uncertainties that are completely uncorrelated, and equation 7.4 corresponds to measurement method uncertainties that are completely correlated, due to common parameters with the same uncertainty. Generally, as more measurements are taken, the contribution of the sampling variance, $\sigma^2_{S,}$ to the overall variance of the mean tends to disappear, whereas some or all of the measurement method variance, $\sigma^2_{M,}$ may remain. The special case where 100% of the M&E is measured may be regarded as the limit when *N* approaches infinity. Some or all of the measurement method variance may still remain.

Once σ is estimated, the power $(1-\beta)$ of a study will depend upon:

1. The Type I decision error rate (α),
2. The size of the gray region (Δ), and
3. The number of measurements made (*N*).

The gray region Δ is the range of radionuclide concentrations or quantities between the DL and the AL. In other words, differences between the DL and the AL less than Δ will be detected with power less than the required $1-\beta$ and therefore are uncertain, or "gray." If the AL is defined as the upper bound of the gray region (UBGR), then the lower bound of the gray region (LBGR) is the DL, and is determined by subtracting Δ from the AL.

All of these factors are interdependent. Generally, the process begins with a known AL, and a DL based on process knowledge. With an estimate of σ, an appropriate number of

measurements, *N*, is found to fulfill the desired limits on decision error rates α and β. If any of these are changed, it will affect the others.

In MARSAME, the null and alternative hypotheses concern the true difference in the M&E between containing radionuclide concentrations or radioactivity in excess of the AL above the appropriate background reference M&E.[1] Scenario A uses a null hypothesis that assumes the radionuclide concentration or radioactivity associated with the M&E exceeds the AL. Scenario A is sometimes referred to as "presumed not to comply" or "presumed not clean." Scenario B uses a null hypothesis that assumes the radionuclide concentration or radioactivity associated with the M&E is less than or equal to the AL. Scenario B is sometimes referred to as "indistinguishable from background" (when the AL is zero) or "presumed clean."

Note: Under Scenario A, the M&E are only deemed suitable for release if the null hypothesis is rejected, whereas under Scenario B, the M&E are suitable for release only if the null hypothesis is not rejected.

For example, under Scenario A, if the true, but unknown, value of the radionuclide concentration or radioactivity in excess of background is less than or equal to the DL, then the hypothesis test upon which the survey is designed will have power $1-\beta$ to reject the null hypothesis that the true, but unknown, value is greater than or equal to the AL at Type I error rate α. Under Scenario B, if the true, but unknown, value of radionuclide concentration or radioactivity in excess of background is greater than the DL (AL + Δ), then the hypothesis test upon which the survey is designed will once again have power $1-\beta$ to reject this null hypothesis at Type I error rate α.

For a given α and $1-\beta$, Δ depends on σ, so it is important that the measurement method (and sampling fraction, where appropriate) selected is sensitive enough to provide a small enough σ, in order that Δ meets survey design requirements for the DL. This ensures that the DL is not set too low in Scenario A or too high in Scenario B. For normally distributed measurements.

$$\Delta/\sigma = (z_{1-\beta} + z_{1-\alpha}) \tag{7-5}$$

Segregation according to likely radionuclide concentrations or radioactivity or a measurement method with a longer counting time may improve σ and therefore Δ. Hypothesis testing (i.e., accepting or rejecting the null hypothesis) consists of comparing an estimate of the radionuclide concentration or radioactivity to a "critical value," S_C. The result indicates whether the observed estimate is consistent with the null value for a given Type I error rate α, after taking account of the uncertainty σ of the measurement. For Scenario A, the critical value is

$$S_C = AL - z_{1-\alpha}\,\sigma \tag{7-6}$$

And for Scenario B the critical value is

$$S_C = AL + z_{1-\alpha}\sigma \tag{7-7}$$

[1] Note that the radionuclides of concern may not be contained in the background reference M&E. If radionuclide specific measurements are made, background reference data will be unnecessary.

Where $z_{1-\alpha}$ is the $1-\alpha$ quantile of the standard normal distribution. In situations where the distribution of the estimate may not be normally distributed, more specialized statistical analysis may be needed. By definition, the power $1-\beta$ is the probability as computed under the alternate hypothesis of rejecting the null hypothesis, or that the probability that the observed estimate is less than the critical value S_C for Scenario A, and greater than S_C for Scenario B.

7.2 Statistical Decision-Making

In Section 4.2, MARSAME recommends the planning team complete the following steps:

- Select a null hypothesis,
- Choose a discrimination limit,
- Define Type I and Type II decision errors,
- Set a tolerable Type I decision error rate at the action level, and
- Set a tolerable Type II decision error rate at the discrimination limit.

7.2.1 Null Hypothesis

In hypothesis testing, two assertions about the actual level of radioactivity associated with the M&E are formulated. The two assertions are called the null hypothesis (H_0) and the alternative hypothesis (H_1). H_0 and H_1 together describe all possible radionuclide concentrations or levels of radioactivity under consideration. The survey data are evaluated to choose which hypothesis to reject or not reject, and by implication which to accept.[2] In any given situation, one and only one of the hypotheses must be true. The null hypothesis is assumed to be true within the established tolerance for making decision errors (Section 7.2.5). Thus, the choice of the null hypothesis also determines the burden of proof for the test.

If the action level (AL) is not zero, the planning team generally assumes the radionuclide concentration or level of radioactivity (X) exceeds the action level unless the survey results provide evidence to the contrary. In other words, surveys are designed to provide sufficient evidence to disprove H_0. In this case, the null hypothesis is that the radionuclide concentration or level of radioactivity is greater than or equal to the action level (i.e., H_0: $X \geq$ AL). The alternative hypothesis is the radionuclide concentration or level of radioactivity is less than the action level (i.e., H_1: $X <$ AL). MARSSIM and NUREG-1505 (NRC 1998a) describe this as Scenario A, and the burden of proof falls on the owner of the M&E. Scenario A is sometimes referred to as "presumed not to comply" or "presumed not clean."

On the other hand, the planning team may choose to assume the action level has not been exceeded unless the survey results provide evidence to the contrary. The null hypothesis becomes H_0: $X \leq$ AL, and the alternative hypothesis is H_1: $X >$ AL. MARSSIM and NUREG-1505 (NRC 1998a) describe this as Scenario B, and the burden of proof falls on the regulator. Scenario B is sometimes referred to as "indistinguishable from background" or "presumed

[2] In hypothesis testing, to "accept" the null hypothesis only means not to reject it. For this reason many statisticians avoid the word "accept." A decision not to reject the null hypothesis does not imply the null hypothesis has been shown to be true.

clean." This is the only practical approach when the action level is equal to zero (above background); because it is technically impossible to obtain statistical evidence that the radionuclide concentration or level of radioactivity is exactly zero. However, Scenario B can be applied to situations other than "indistinguishable from background." The example in Section 8.4 uses Scenario B to support an interdiction decision.

7.2.2 Discrimination Limit

Action levels were defined in Section 3.3 based on the selected disposition option and applicable regulatory requirements. The planning team also chooses another radionuclide concentration or level of radioactivity that can be reliably distinguished from the action level by performing measurements (i.e., direct measurements, scans, in situ measurements, samples and laboratory analyses). This radionuclide concentration or level of radioactivity is called the discrimination limit (DL). An example where the discrimination limit is defined is provided in Section 8.4.5.The gray region is defined as the interval between the action level and the discrimination limit (Figures 7.1, 7.2, 7.3, and 7.4 provide visual descriptions of the gray region). The width of the gray region is called the shift and denoted as Δ. The objective of the disposition survey is to decide whether the concentration of radioactivity is more characteristic of the DL or of the AL, i.e., whether action should be taken, or if action is not necessary. Figures 7.1 and 7.2 show examples that would fall under Scenario A (discussed in Section 7.2.3). In Figure 7.1 (top) the difference in concentration between the AL and the DL (i.e., Δ) is large; but the variability in the measured concentration (i.e., σ) is also large. In Figure 7.2 (bottom) the difference in concentration between the AL and the DL (i.e., Δ) is relatively small. However, the variability in the measured concentration (i.e., σ) is also smaller. Figures 7.1 and 7.2 illustrate that determining the level of survey effort depends not just on the width of the gray region, but also in the ratio of that width to the expected variability of the data. This ratio, Δ/σ, is called the relative shift in MARSSIM. In situations where Δ/σ is small, i.e., less than 1, it may be impracticable to achieve the required accuracy of measurements or the number of samples to meet the Type I error rate in the DQOs. Section 4.4.4 presents options for relaxing project constraints to optimize the survey design in such cases. In Figure 7.1, Δ/σ is greater than 4; while in Figure 7.2, Δ/σ is approximately 1.

As discussed in MARSSIM, generally, the larger Δ/σ, the easier the survey effort. When Δ/σ is greater than three, the survey effort will be minimal, and any effort to increase it by either widening the gray region or reducing the measurement variability usually would not be worthwhile.

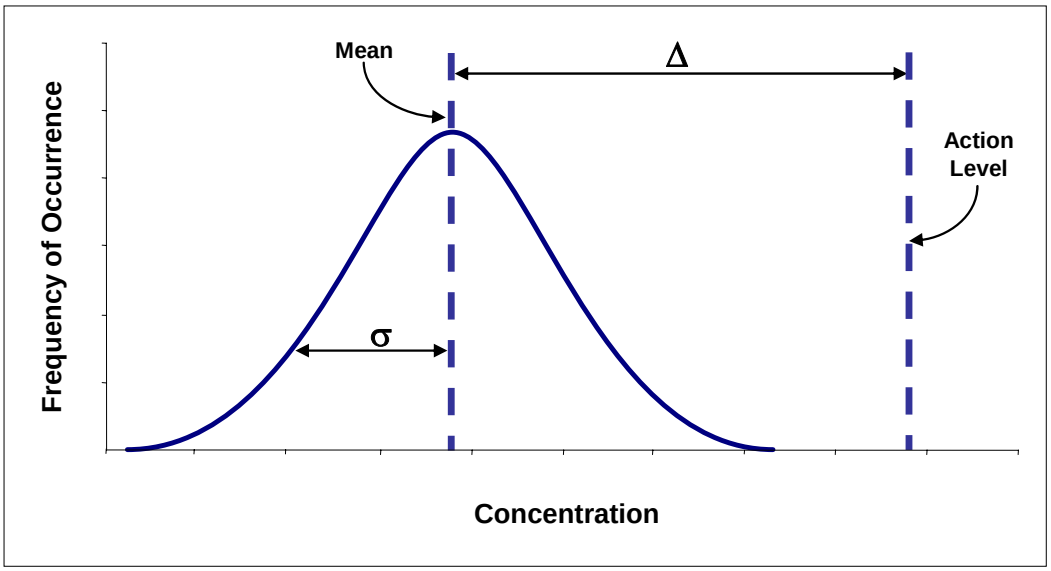

Figure 7.1 Relative Shift, Δ/σ, Comparison for Scenario A:
σ is Large, but the Large Δ Results in a Large Δ/σ and Fewer Samples

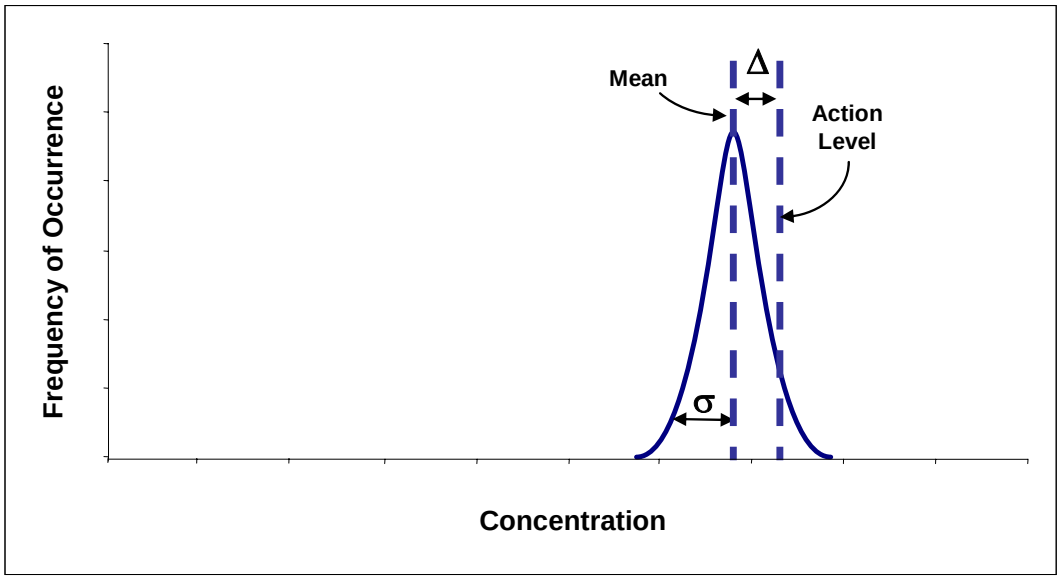

Figure 7.2 Relative Shift, Δ/σ, Comparison for Scenario A:
σ is Small, but the Small Δ Results in a Small Δ/σ and More Samples

On the other hand, when Δ/σ is less than one, the survey effort will become substantial, and any effort to increase it by either widening the gray region or reducing the measurement variability will be worthwhile. The measurement variability is thus just as important as the width of the gray region when designing disposition surveys. In MARSSIM surveys, the total variability had two components: sampling and analytical. For some MARSAME surveys this will also be the case. However, in many MARSAME surveys the sampling variability will be of less importance, either because 100% of the survey unit is being measured, or because disposition decisions are being made on the basis of single measurements on single items or single locations. In such

cases, the required measurement method uncertainty discussed in Section 3.8.1 will be of paramount importance in the survey planning. The details for determining the required measurement method uncertainty and how to determine if it is being met are discussed in detail in Section 7.7.

Depending on the survey design, the combination of action levels, expected radionuclide concentrations or levels of radioactivity, instrument sensitivity, and local radiation background contribute to defining the width of the gray region. Reducing the radionuclide concentrations or levels of radioactivity known or assumed to be associated with the M&E can affect the selection of a discrimination limit, so remediation costs may need to be considered. Increasing the sensitivity of a measurement method to reduce the measurement method uncertainty generally involves increased instrument costs or increased counting times.

The lower bound of the gray region is denoted by LBGR and the upper bound of the gray region is denoted by UBGR. The association of either the UBGR or the LBGR with the DL or AL will depend on the scenario selected (see Sections 7.2.3 and 7.2.4). The width of the gray region (UBGR – LBGR) is denoted by "Δ" and is called the "shift" or the "required minimum detectable difference" in activity or concentration (MARSSIM Section 5.5.2 and Section D.6, MARLAP Section C.2, NRC 1998a, and EPA 2006a,).

7.2.3 Scenario A

The null hypothesis for Scenario A specifies that the radionuclide concentration or level of radioactivity associated with the M&E is equal to or exceeds the action level. For Scenario A (H_0: $X \geq$ AL), the UBGR is equal to the AL and the LBGR is equal to the DL. As a general rule for applying Scenario A, the DL should be set no higher than the expected radionuclide concentration associated with the M&E. The DL and the AL should be reported in the same units. Figure 7.3 illustrates Scenario A. Note that the Type I (α) and Type II (β) error rates need not be equal. This is discussed further in Section 7.2.5, and an example can be seen in Section 7.5.2.

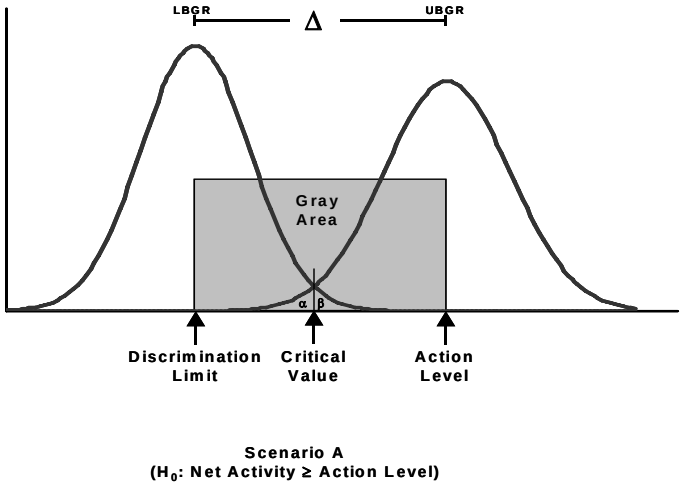

Figure 7.3 Illustration of Scenario A

7.2.4 Scenario B

The null hypothesis for Scenario B specifies the radionuclide concentration or level of radioactivity associated with the M&E is less than or equal to the action level. For Scenario B (H_0: $X \leq$ AL), the UBGR is equal to the DL and the LBGR is equal to the AL. For example, if the AL=0 (sometimes called indistinguishable from background), then the LBGR will be zero. The DL defines how hard the surveyor needs to look, and is determined through negotiations with the regulator.[3] In some cases, the DL will be set equal to a regulatory limit (e.g., 10 CFR 36.57 and DOE 1993). The DL and the AL should be reported in the same units. Figure 7.4 illustrates Scenario B. As above, note that the Type I (α) and Type II (β) error rates need not be equal. This is discussed further in Section 7.2.5, and an example can be seen in Section 7.5.2.

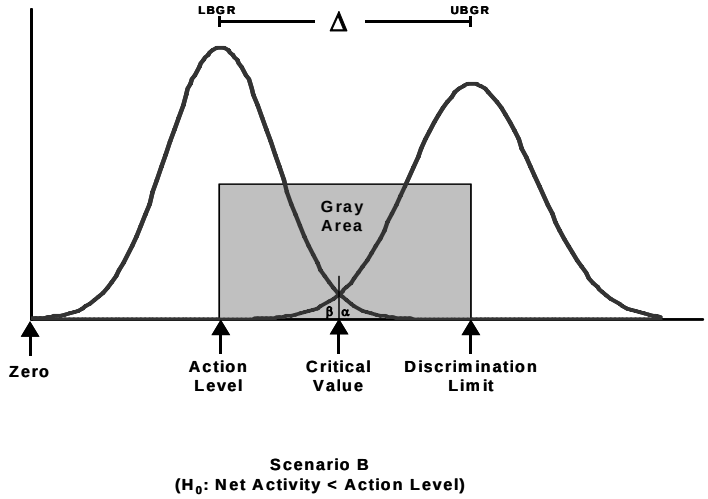

Figure 7.4 Illustration of Scenario B

This description of Scenario B is based on information in MARLAP and is fundamentally different from the description of Scenario B in NUREG-1505 (NRC 1998a).

In NUREG-1505 (NRC 1998a) the gray region is defined as being below the AL in both Scenario A and Scenario B. In MARSAME and MARLAP the gray region is defined as being above the AL in Scenario B. The difference lies in how the action level is defined.

7.2.5 Specify Limits on Decision Errors

There are two possible types of decision errors:

- Type I error: rejecting the null hypothesis when it is true.
- Type II error: failing to reject the null hypothesis when it is false.

[3] In some cases setting the discrimination limit may include negotiations with stakeholders.

Because there is always uncertainty associated with the survey results, the possibility of decision errors cannot be eliminated. So instead, the planning team specifies the maximum Type I decision error rate (α) that is allowable when the radionuclide concentration or level of radioactivity is at or above the action level. This maximum usually occurs when the true radionuclide concentration or level of radioactivity is exactly equal to the action level. The planning team also specifies the maximum Type II decision error rate (β) that is allowable when the radionuclide concentration or level of radioactivity equals the discrimination limit. Equivalently, the planning team can set the "power" ($1-\beta$) when the radionuclide concentration or level of radioactivity equals the discrimination limit. See MARSSIM Appendix D, Section D.6, for a more detailed description of error rates and statistical power.

The definition of decision errors depends on the selection of the null hypothesis. For Scenario A the null hypothesis is the radionuclide concentration or level of radioactivity exceeds the action level. A Type I error for Scenario A occurs when the decision maker decides the radionuclide concentration or level of radioactivity is below the action level when it is actually above the action level (i.e., mistakenly decides the M&E are clean when they are actually not clean). A Type II error for Scenario A occurs when the decision maker decides the radionuclide concentration or level of radioactivity is above the action level when it is actually below the action level (i.e., mistakenly decides the M&E are not clean when they are actually clean).

For Scenario B, the null hypothesis is that the radionuclide concentration or level of radioactivity is less than or equal to the action level. A Type I error for Scenario B occurs when the decision maker decides the radionuclide concentration or level of radioactivity is above the action level when it is actually below the action level (i.e., mistakenly decides the M&E are not clean when they are actually clean). A Type II error for Scenario B occurs when the decision maker decides the radionuclide concentration or level of radioactivity is below the action level when it is actually above the action level (i.e., mistakenly decides the M&E are clean when they are actually not clean). It is important to clearly define the scenario (i.e., A or B) and the decision errors for the survey being designed.

Once the decision errors have been defined, the planning team should determine the consequences of making each type of decision error. This process should be revisited as more information is obtained. For example, incorrectly deciding the activity is less than the action level may result in increased health and ecological risks. Incorrectly deciding the activity is above the action level when it is actually below may result in increased economic and social risks. The consequences of making decision errors are specific to the actual situation at a particular site and could vary significantly from one site to another, reflecting the major concerns of the various stakeholders.

Once the consequences of making both types of decision errors have been identified, acceptable decision error rates can be assigned for both Type I and Type II decision errors. Historically, a decision error rate of 0.05, or 5%, often has been acceptable for decision errors. However, assigning the same tolerable decision error rate to all projects does not account for the differences in consequences of making decision errors. This becomes evident with M&E where there are wide ranges of disposition options generating a wide range of consequences. For example, a Type I decision error for Scenario A could have different consequences for a

clearance decision compared to a low-level radioactive waste disposal decision. Not all consequences of decision errors are the same, and it is unlikely that applying a fixed value to all decision error rates will result in reasonable survey designs resulting in comparable decisions. Project-specific decision error rates should be selected based on the project-specific consequences of making decision errors.

7.2.6 Develop an Operational Decision Rule

The theoretical decision rule developed in Section 3.7 was based on the assumption that the true radioactivity concentrations or radiation levels associated with the M&E were known. Since the disposition decision will be made based on measurement results and not the true but unknown concentration level, an operational decision rule needs to be developed to replace the theoretical decision rule. The operational decision rule is a statement of the statistical hypothesis test, which is based on comparing some function of the measurement results to some critical value. The theoretical decision rule is developed during Step 5 of the DQO Process (Chapter 3), while the operational decision rule is developed as part of Step 6 and Step 7 of the DQO Process. For example, a theoretical decision rule might be "if the results of any measurement identify surface radioactivity in excess of background, the front loader will be refused access to the site; if no surface radioactivity in excess of background is detected, the front loader will be granted access to the site." The related operational decision rule might be "any result that exceeds the critical value associated with the MDC, set at the discrimination limit, will result in rejection of the null hypothesis and the front loader will not be allowed on the site" (see more examples in Chapter 8).

Chapter 6 provides guidance on using statistical tests to evaluate data collected during the disposition survey to support a disposition decision. The planning team should evaluate the statistical tests and possible operational decision rules and select one that best matches the intent of the theoretical decision rule with the statistical assumptions. Each operational decision rule will have a different formula for determining the number of measurements or fraction of M&E to be measured to meet the DQOs.

Developing an operational decision rule incorporates all relevant information available concerning the M&E (Section 2.4.3), selected instrumentation and measurement technique (Section 5.9), selected statistical tests (Section 6.2.3), and any constraints on collecting data identified by the planning team. The operational decision rule will need to specify a measurement technique (e.g., scan-only, in situ, sample collection and analysis) and a statistical test. Examples of statistical tests include comparison to the UBGR (Section 6.3), comparison to an upper confidence interval (Section 6.4), the Sign test (Section 6.5), the Wilcoxon Rank Sum test (Section 6.6), and the Quantile test (Section 6.7). At this point in the survey design process it is not necessary to select a specific instrument to perform the measurements. However, selection of a measurement technique will assist the planning team in identifying the appropriate statistical test. For example, if a scan-only measurement method is selected it is not appropriate to select the Wilcoxon Rank Sum test to determine the number of measurements. However, if no scan-only or in situ measurement methods are available that meet the measurement quality objectives (MQOs), a MARSSIM-type survey (which combines scan and static measurements, see Section 4.4.3) should be developed.

The planning team uses the combination of the selected instrumentation and measurement technique (see Section 5.9) with a data evaluation method (see Section 6.2.5) to establish an operational decision rule. Then, from the operational decision rule, the planning team can determine the number of measurements or the fraction of the M&E that needs to be measured during the disposition survey. There is no formal structure for stating an operational decision rule. The structure of the operational decision rule is generally defined in terms that meet the needs of a particular project. An operational decision rule can be simple or complex. A simple example could be "If 100% of the surfaces of hand tools are surveyed using a scan-only technique that meets the DQOs, and none of the results exceed the action level for release, then the tools can be released." The statistical test for this simple example is a comparison of the mean to the action level; however, since all of the values are below the action level, the mean value must also be below the action level. Therefore, it is not necessary to perform the actual statistical test. This represents a conservative approach to data interpretation that may not always be appropriate. More complex operational decision rules can–

- Account for different types of measurements and multiple radionuclides of concern,
- Specify critical values and test statistics for the statistical tests, and
- Incorporate multiple decisions (e.g., average and maximum values, fixed and removable radioactivity) depending on the project.

7.3 Set Measurement Quality Objectives

Section 4.2 briefly discussed the DQO process for developing statistical hypothesis tests for the implementation of disposition decision rules using measurement data. This included formulating the null and alternative hypotheses, defining the gray region using the action level and discrimination limit, and setting the desired limits on potential Type I and Type II decision error probabilities that a decision maker is willing to accept for project results. Decision errors are possible, at least in part, because measurement results have uncertainties. The effect of these uncertainties is expressed in the size of the relative shift, Δ/σ, introduced in Section 7.2.2. The overall uncertainty, σ, has components that may be due to sampling variability in radioactivity concentration, σ_S, but also because of uncertainty in the measurement method, σ_M. Because DQOs apply to both sampling and measurement activities, what are needed from a measurement perspective are method performance characteristics specifically for the measurement process of a particular project. These method performance characteristics (see Section 3.8) are the measurement quality objectives (MQOs).

DQOs define the performance criteria that limit the probabilities of making decision errors by–

- Considering the purpose of collecting the data,
- Defining the appropriate type of data needed, and
- Specifying tolerable probabilities of making decision errors.

DQOs apply to both sampling and measurement activities.

MQOs can be viewed as the measurement portion of the overall project DQOs (see Section 3.8). MQOs are:

- The part of the project DQOs that apply to the measured result and its associated uncertainty.
- Statements of measurement performance objectives or requirements for a particular measurement method performance characteristic, for example, measurement method uncertainty and detection capability.
- Used initially for the selection and evaluation of measurement methods.
- Subsequently used for the ongoing and final evaluation of the measurement data.

A number of MQOs were introduced in Section 3.8, but for survey planning the single most important MQO is the required measurement method uncertainty, u_{MR}. Other MQOs, such as range, ruggedness, and specificity, if not controlled, will lead to increased measurement uncertainty. In this sense, the required measurement method uncertainty encompasses many of the effects of other MQO parameters that could impact decision making. MDCs and MQCs are closely related to the measurement uncertainty, have a long history of use for comparing the appropriateness of competing measurement techniques, and can contribute much to survey planning. These concepts are developed further in the later sections of this chapter (Sections 7.5 and 7.6). However, essentially the same information can be conveyed by specifying the required measurement method uncertainty, which is a more general concept. Thus, in this section and the next, it is this MQO that will be emphasized.

Measurement method uncertainty refers to the *predicted* uncertainty of a measured value that would be calculated if the method were applied to a hypothetical sample with a specified radioactivity concentration or radiation level. Measurement method uncertainty is a characteristic of the measurement method and the measurement process. Measurement uncertainty, as opposed to sampling uncertainty, is a characteristic of an individual measurement.

The true measurement method standard deviation, σ_M, is a theoretical quantity and is never known exactly, but it may be estimated using the methods described in Section 7.4. The estimate of σ_M will be denoted here by u_M and called the "measurement method uncertainty." The measurement method uncertainty, when estimated by uncertainty propagation, is the predicted value of the combined standard uncertainty (CSU, or "one-sigma" uncertainty) of the measurement for material with concentration equal to the UBGR. Note that the term "measurement method uncertainty" and the symbol u_M actually apply not just to the measurement method but also to the entire measurement process, that is, it should include uncertainties in how the measurement method is actually implemented. This definition of measurement method uncertainty is independent of the null hypothesis and applies to both Scenario A and Scenario B.

The true standard deviation of the measurement method, σ_M, is unknown, but the required measurement method uncertainty, σ_{MR}, is intended to be an upper bound for σ_M. In practice, σ_{MR} is actually used as an upper bound for the method uncertainty, u_M, which is an estimate of σ_M. Therefore, the value of σ_{MR} will be called the "required measurement method uncertainty" and denoted by u_{MR}.

The principal MQOs in any project will be defined by the required measurement method uncertainty, u_{MR}, at and below the UBGR and the relative required measurement method uncertainty, φ_{MR}, at and above the UBGR, $\varphi_{MR} = u_{MR}$/UBGR. See Section 7.3.2 for further discussion.

When making decisions about individual measurement results u_{MR} should ideally be 0.3Δ, and when making decisions about the mean of several measurement results u_{MR} should ideally be 0.1Δ, where Δ is the width of the gray region, Δ = UBGR – LBGR.

7.3.1 Determine the Required Measurement Method Uncertainty at the UBGR

This section provides the rationale and guidance for establishing project-specific MQOs for controlling σ_M. This control is achieved by establishing a desired maximum measurement method uncertainty, u_{MR}, at the upper boundary of the gray region. This control also will assist in both the measurement method selection process and in the evaluation of measurement data. Approaches applicable to several situations are detailed below.

Four basic survey designs were described in Chapter 4: scan-only, in situ, MARSSIM-type, and method-based. The relative shift, Δ/σ, is important in determining the level of survey effort required in the first three survey designs. For a given width of the gray region, Δ, the relative shift, Δ/σ, can only be controlled by controlling σ. The overall standard deviation of the measurement results, σ, may have both a measurement component, σ_M, and a sampling component, σ_S. Segregation and classification may help in controlling σ_S (Sections 4.3 and 5.4).

7.3.1.1 Scan-Only Survey Designs

For 100% scan-only surveys, the decision uncertainty associated with σ_S is essentially eliminated because the entire survey unit is measured. In class 2 survey units, the scan coverage can vary from 10% to nearly 100% depending on the value of Δ/σ. This is a reflection of the fact that for a fixed measurement variability, σ_M, smaller values of Δ/σ imply larger sampling variability. Larger sampling variability demands higher scan coverage to reduce the decision uncertainty. That is, more of the survey unit must be measured to lower the standard deviation of the mean. In such cases, it will be desirable to reduce σ_M until it is negligible in comparison to σ_S. σ_M can be considered negligible if it is no greater than $\sigma_S/3$. Therefore, MARSAME recommends the requirement $u_{MR} \leq \sigma_S/3$.

7.3.1.2 In Situ Survey Designs

For in situ survey designs, either the entire survey unit, or a large portion of it (e.g., greater than 10%), is covered with a single measurement. Thus, sampling variability will tend to be averaged out. When decisions are to be made by comparing such single measurements to an action level, the total variance of the data equals the measurement variance, σ_M^2, and the data distribution in most instances should be approximately normal. In these cases the DQOs will be met if

$$u_{MR} \leq \frac{\text{UBGR-LBGR}}{z_{1-\alpha} + z_{1-\beta}} = \frac{\Delta}{z_{1-\alpha} + z_{1-\beta}} \tag{7-8}$$

where $z_{1-\alpha}$, is the $(1 - \alpha)$-quantile of the standard normal distribution and $z_{1-\beta}$, is the $(1 - \beta)$-quantile of the standard normal distribution.

If $\alpha = \beta = 0.05$, then

$$u_{Mr} \le \frac{\Delta}{z_{0.95} + z_{0.95}} = \frac{\Delta}{1.645 + 1.645} = \frac{\Delta}{3.29} \sim 0.3\,\Delta \qquad \textbf{(7-9)}$$

Therefore, MARSAME recommends the requirement $u_{MR} \le 0.3\Delta$. The details are discussed in Section 7.7.2.

For the special case where the LBGR = 0, then Δ = UBGR and $\sigma_{MR} = \Delta / (z_{1-\alpha} + z_{1-\beta})$ implies

$$u_{MR} \le \frac{UBGR}{z_{0.95} + z_{0.95}} = \frac{UBGR}{1.645 + 1.645} = \frac{UBGR}{3.29} \sim 0.3\,UBGR \qquad \textbf{(7-10)}$$

This is equivalent to requiring that the MDC (see Section 7.9.2) be less than the action level. The MDC is defined as the concentration at which the probability of detection is $1 - \beta$ and the probability of false detection in a sample with zero concentration is at most α.

Example 1: Suppose the action level is 10,000 Bq/m^2 and the lower bound of the gray region is 5,000 Bq/m^2, $\alpha = 0.05$, and $\beta = 0.10$. If decisions are to be made about individual items, then the required measurement method uncertainty at 10,000 Bq/m^2 is

$$u_{MR} = \frac{\Delta}{z_{1-\alpha} + z_{1-\beta}} = \frac{10{,}000 \text{ Bq/m}^2 - 5{,}000 \text{ Bq/m}^2}{z_{0.95} + z_{0.90}} = \frac{5{,}000 \text{ Bq/m}^2}{1.645 + 1.282} = 1{,}700 \text{ Bq/m}^2$$

7.3.1.3 MARSSIM-Type Survey Designs

When a decision is to be made about the mean of a sampled population, generally the average of a set of measurements on a survey unit is compared to the disposition criterion. For MARSSIM-type designs, the ratio Δ/σ, called the "relative shift," determines the number of measurements required to achieve the desired decision error rates α and β. The target range for this ratio should be between 1 and 3, as explained in MARSSIM (MARSSIM 2002) and NUREG-1505 (NRC 1998a). Ideally, to keep the required number of measurements low, the DQOs are aimed at establishing $\Delta/\sigma \approx 3$. The cost in number of measurements rises rapidly as the ratio Δ/σfalls below 1, but there is little benefit from increasing the ratio much above 3. One of the main objectives in optimizing survey design is to achieve a relative shift, Δ/σ, of at least one and ideally three. Values of Δ/σ greater than three, while desirable, should not be pursued at additional cost. If Δ/σ is 3 and σ_M is negligible in comparison to σ_S, then σ_M will be $\Delta/10$. The details are discussed in Section 7.7.1.

Therefore, MARSAME recommends the requirement $u_{MR} \le \Delta / 10$ by default when decisions are being made about the mean of a sampled population. If the LBGR is zero, this is equivalent to requiring that the MQC be less than the UBGR (Section 7.7.1).

Example 2: Suppose the action level is 10,000 Bq/m^2 and the lower bound of the gray region is 2,000 Bq/m^2. If decisions are to be made about survey units based on measurements at several locations, then the required measurement method uncertainty (u_{MR}) at 10,000 Bq/m^2 is

$$\mu_{MR} = \frac{\Delta}{10} = \frac{10,000 - 2,000}{10} = 800 \text{ Bq/m}^2$$

Example 3: Suppose the action level is 10,000 Bq/m^2, but this time assume the lower bound of the gray region is 0 Bq/m^2. In this case the required method measurement uncertainty, u_{MR}, at 10,000 Bq/m^2 is

$$\mu_{MR} = \frac{\Delta}{10} = \frac{10,000 - 0}{10} = 1,000 \ Bq/m^2$$

The recommended values of u_{MR} are based on the assumption that any known bias in the measurement process has been corrected and that any remaining bias is well less than 10% of the shift, Δ, when a concentration near the gray region is measured.

Achieving a required measurement method uncertainty u_{MR} less than the recommended limits may be difficult in some situations. When the recommended requirement for u_{MR} is too difficult to meet, project planners may allow u_{MR} to be larger. In this case, project planners may choose u_{MR} to be as large as $\Delta/3$ or any calculated value that allows the data quality objectives to be met at an acceptable effort. Two situations that may make this possible are if σ_S is believed to be less than $\Delta/10$ or if it is not difficult to make the additional measurements required by the larger overall data variance ($\sigma_M^2 + \sigma_S^2$).

Example 4: Suppose the uncertainty in Example 2 of u_{MR} = 800 Bq/m^2 cannot be achieved because of the variability in instrument efficiency with surface roughness. A required measurement method uncertainty, u_{MR}, as large as $\Delta / 3 \approx$ 2,700 Bq/m^2 may be possible if σ_S is small or if more measurements are taken per survey unit.

7.3.2 Determine the Required Measurement Method Uncertainty at Concentrations Other Than the UBGR

The most important MQO for data evaluation is the one for measurement method uncertainty at a specified concentration. This MQO is expressed as the required measurement method uncertainty (u_{MR}) at the UBGR. However, to properly evaluate the data usability of measurement results at concentrations other than the UBGR, the implications of this requirement must be extended both above and below the UBGR.

When the concentration is less than or equal to the UBGR, the combined standard uncertainty (CSU), u_c, of a measured result should not exceed the required measurement method uncertainty, u_{MR}, specified at the UBGR. When the concentration is greater than the UBGR, the relative combined standard uncertainty (RCSU), φ_{MR}, of a measured result should not exceed the required relative measurement method uncertainty at the UBGR.

$$\varphi_{MR} = u_{MR}/\text{UBGR} \tag{7.11}$$

This is illustrated in Example 5 and Figure 7.5.

Example 5: Suppose the action level is 10,000 Bq/m^2 and the discrimination limit is 3,000. Scenario A is used, so the UBGR = AL = 10,000 Bq/m^2 and the LBGR = DL = 3,000 Bq/m^2. Thus the width of the gray region, Δ= 10,000 – 3,000 = 7,000. If decisions are to be made about individual items, α = 0.05, and β = 0.05, then the required measurement uncertainty at 10,000 Bq/m^2 is

$$u_{MR} = \frac{\Delta}{z_{1-\alpha} + z_{1-\beta}} = \frac{10{,}000 \text{ Bq/m}^2 - 3{,}000 \text{ Bq/m}^2}{z_{0.95} + z_{0.95}} = \frac{7{,}000 \text{ Bq/m}^2}{1.645 + 1.645} \approx 2{,}000 \text{ Bq/m}^2$$

The required measurement method uncertainty, u_{MR}, is 2,000 Bq/m^2 at 10,000 Bq/m^2. Thus, for any measured result less than 10,000 Bq/m^2, the reported CSU, u_c, should be less than or equal to 2,000 Bq/m^2. For example, a reported result of 4,500 Bq/m^2 with a CSU of 1,900 Bq/m^2 would meet the requirement. A reported result of 7,700 Bq/m^2 with a CSU 2,500 Bq/m^2 would not meet the requirement.

The required relative measurement method uncertainty (φ_{MR}) is 2,000 Bq/m^2 / 10,000 Bq/m^2 = 20% at 10,000 Bq/m^2. Thus, for any measured result greater than 10,000 Bq/m^2, the reported RCSU should be less than or equal to 20%. For example, a reported result of 14,500 Bq/m^2 with a CSU of 2,900 Bq/m^2 would meet the requirement because 2,900/14,500 = 20%. A reported result of 18,000 Bq/m^2 with a CSU 4,500 Bq/cm^2 would not meet the requirement because 4,500/18,000 = 25%.

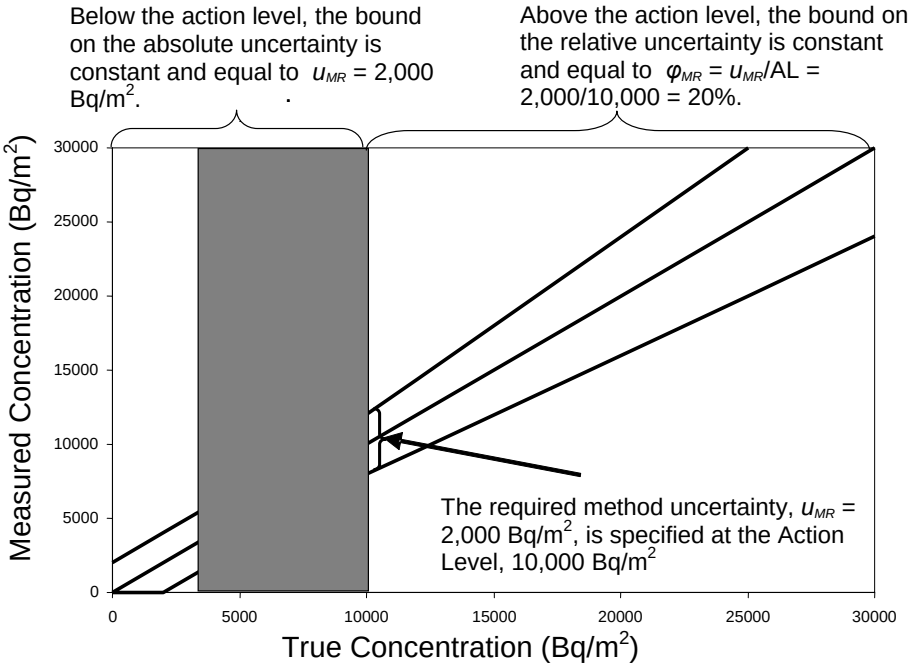

Figure 7.5 Example of the Required Measurement Uncertainty at Concentrations other than the UBGR. In this Example the UBGR Equals the Action Level

7.4 Determine Measurement Uncertainty

Checking the measurement quality against the required measurement method uncertainty relies on having realistic estimates of the measurement uncertainty. Often reported measurement uncertainties are underestimated, particularly if they are confined to the estimated Poisson counting uncertainty (Section 7.8). Tables of results are sometimes presented with a column listing "±" without indicating how these numbers were obtained. Often, the "±" represents the square root of the number of counts obtained during the measurement. The method for evaluation calculation and reporting of measurement uncertainty, approved by both the International Organization for Standardization (ISO) and the National Institute of Standards and Technology (NIST) is discussed in this section. Further details of the method are given in Section 7.8.

Measurements always involve uncertainty, which must be considered when measurement results are used as part of a basis for making decisions. Every measured and reported result should be accompanied by an explicit uncertainty estimate. One purpose of this section is to give users of data an understanding of the causes of measurement uncertainty and of the meaning of uncertainty statements; another is to describe procedures that can be used to estimate uncertainties. Much of this material is derived from MARLAP Chapter 19.

In 1980, the Environmental Protection Agency published a report entitled *Upgrading Environmental Radiation Data*, which was produced by an ad hoc committee of the Health Physics Society (EPA 1980). Two of the recommendations of this report were that:

1. Every reported measurement result (x) should include an estimate of its overall uncertainty (u_x) that is based on as nearly a complete an assessment as possible.
2. The uncertainty assessment should include every significant source of inaccuracy in the result.

The concept of traceability is also defined in terms of uncertainty. Traceability is defined as the "property of the result of a measurement or the value of a standard whereby it can be related to stated references, usually national or international standards, through an unbroken chain of comparisons all having stated uncertainties" (ISO 1996). Thus, to realistically make the claim that a measurement result is "traceable" to a standard, there must be a chain of comparisons (each measurement having its own associated uncertainty) connecting the result of the measurement to that standard.

This section considers only measurement variability, σ_M. Reducing sampling variability, σ_S, by segregating M&E was discussed in Section 5.4. Sampling variability due to field sampling uncertainties is often larger than measurement uncertainties. Although this statement may be true in some cases, this is not an argument for failing to perform a full evaluation of the measurement uncertainty. A realistic estimate of the measurement uncertainty is one of the most useful data quality indicators for a result (Section 3.8).

Although the need for reporting uncertainty has sometimes been recognized, often it consists of only the estimated component due to Poisson counting statistics. The component of uncertainty

resulting from the random nature of radioactive decay is only one component of measurement method uncertainty. If only this component of uncertainty is accounted for, rather than performing a full uncertainty analysis, the result will be misleading because it is at best only a lower bound of the uncertainty and may lead to incorrect decisions based on overconfidence in the measurement. Software is available to perform the mathematical operations for uncertainty evaluation and propagation, eliminating much of the difficulty in implementing the mathematics of uncertainty calculations. There are several examples of such software (McCroan 2006, GUM Workbench 2006, Kragten 1994, and Vetter 2006).

7.4.1 Use Standard Terminology

The methods, terms, and symbols recommended by MARSAME for evaluating and expressing measurement uncertainty are described in the GUM (ISO 1995). The ISO methodology is summarized in the NIST Technical Note TN-1297 (NIST 1994).

The result of a measurement is generally used to estimate some particular quantity called the measurand. The difference between the measured result and the actual value of the measurand is the error of the measurement. Both the measured result and the error may vary with each repetition of the measurement, while the value of the measurand (the true value) remains fixed. The error of a measurement is unknowable, because one cannot know the error without knowing the true value of the quantity being measured (the measurand). For this reason, the error is primarily a theoretical concept. However, the uncertainty of a measurement is a concept with practical uses. According to the GUM and NIST Technical Note 1297, the term "uncertainty of measurement" denotes the values that could reasonably be attributed to the measurand. In practice, there is seldom a need to refer to the error of a measurement, but an uncertainty should be stated for every measured result.

The first step in defining a measurement process is to define the measurand clearly. The specification of the measurand is always ambiguous to some extent, but it should be as clear as necessary for the intended purpose of the data. For example, when measuring the activity of a radionuclide on a surface, it is generally necessary to specify the activity, the date and time, what area of the surface was measured, and where.

Often the measurand is not measured directly but instead an estimate is calculated from the measured values of other input quantities, which have a known mathematical relationship to the measurand. For example, input quantities in a measurement of radioactivity may include the gross count, blank or background count, counting efficiency, and area measured. The mathematical model measurement process specifies the relationship between the output quantity, Y, and measurable input quantities, $X_1, X_2, \ldots X_N$, on which its value depends: $Y = f(X_1, X_2, \ldots X_N)$.

The mathematical model for a radioactivity measurement may have the simple form:

$$\text{Measurement} = \frac{(\text{Gross Instrument Signal}) - (\text{Blank Signal})}{\text{Efficiency}} \quad \textbf{(7-12)}$$

Each of the quantities shown here may actually be a more complicated expression. For example, the efficiency may be the product of factors such as surveyor efficiency, surface roughness

efficiency correction, and the instrument counting efficiency. Interferences may be due to ambient background or other radionuclides that have interactions with the detector in a manner that contributes spuriously to the gross instrument signal.

When a measurement is performed, a specific value x_i is estimated for each input quantity, X_i, and an estimated value, y, of the measurand is calculated using the relationship $y = f(x_1, x_2,\ldots,x_N)$. Since there is an uncertainty in each input estimate, x_i, there is also an uncertainty in the output estimate, y. Determining the uncertainty of the output estimate y requires that the uncertainties of all the input estimates x_i be determined and expressed in comparable forms. The uncertainty of x_i is expressed in the form of an estimated standard deviation, called the standard uncertainty and denoted by $u(x_i)$. The ratio $u(x_i) / |X_i|$ is called the relative standard uncertainty of x_i, where $|X_i|$ is the absolute value of x_i.

The partial derivatives, $\partial f / \partial x_i$, are called sensitivity coefficients, and are usually denoted by c_i. The c_i measure how much f changes when x_i changes. The standard uncertainties are combined with sensitivity coefficients to obtain the component of the uncertainty in y due to x_i, $c_i u(x_i)$.

The square of the CSU, denoted by $u_c^2(y)$, is called the combined variance. It is obtained using the formula for the propagation of uncertainty:[4]

$$u_c^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) = \sum_{i=1}^{N} c_i^2 u^2(x_i) \qquad \textbf{(7-13)}$$

The square root of the combined variance is the CSU of y, denoted by $u_c(y)$. Further details of this process are given in Section 7.8.1.

7.4.2 Consider Sources of Uncertainty

The following sources of uncertainty should be considered:

- The random nature of radioactive decay (e.g., counting statistics),
- Instrument calibration (e.g., counting efficiency),
- Variable instrument backgrounds,
- Variable counting efficiency (e.g., due to the instrument or to source geometry and placement), and
- Interferences, such as crosstalk and spillover.

Other sources of uncertainty could include:

- Temperature and pressure.
- Volume and mass measurements,
- Determination of counting time and correction for dead time,

[4] If the input estimates are potentially correlated, covariance estimates $u(x_i,x_j)$ must also be determined. The covariance $u(x_i,x_j)$ is often recorded and presented in the form of an estimated correlation coefficient, $r(x_i,x_j)$, which is defined as the quotient $u(x_i,x_j) / u(x_i)u(x_j)$. See Section 7.8.

- Time measurements used in decay and ingrowth calculations,
- Approximation errors in simplified mathematical models, and
- Published values for half-lives and radiation emission probabilities.
-

There are a number of sources of measurement uncertainty in gamma-ray spectroscopy, including:

- Poisson counting uncertainty,
- Compton baseline determination,
- Background peak subtraction,
- Multiplets and interference corrections,
- Peak-fitting model errors,
- Efficiency calibration model error,
- Summing,
- Density-correction factors, and
- Dead time.

Additional discussion of some major sources of uncertainty may be found in Section 7.8.2.2.

The following example may appear complex, but all but the most casual users will use software to perform these calculations. Some possibilities are listed after the example. A complete example is worked out to here to illustrate the underlying principles.

Example 6: Consider a simple measurement of a sample. The activity will be calculated from

$$y = \frac{(N_S / t_S) - (N_B / t_B)}{\varepsilon}$$

Where:

y = sample activity (Bq)
ε = counting efficiency 0.4176 (s^{-1}/Bq)
N_S = gross count observed during the measurement of the source, (11578)
t_S = source count time (300 s)
N_B = observed background count (87)
t_B = background count time (6,000 s)

The CSU of ε is given by $u_C(\varepsilon) = 0.005802$. This is shown in Example 2 in Section 7.8.2.2. Assume the radionuclide is long-lived; so, no decay corrections are needed. The uncertainties of the count times are also assumed to be negligible. The standard uncertainties in N_S and N_B will be estimated as $\sqrt{N_S}$ and $\sqrt{N_B}$ using the Poisson assumption.

Then, $y = \frac{(N_S / t_S) - (N_B / t_B)}{\varepsilon} = \frac{(11578/300) - (87/6000)}{0.4179} = 92.316$

$$u_c^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) = \sum_{i=1}^{N} c_i^2 u^2(x_i)$$

$$= \left(\frac{\partial \frac{(N_S / t_S) - (N_B / t_B)}{\varepsilon}}{\partial N_S} \right)^2 u^2(N_S) + \left(\frac{\partial \frac{(N_S / t_S) - (N_B / t_B)}{\varepsilon}}{\partial N_B} \right)^2 u^2(N_B)$$

$$+ \left(\frac{\partial \frac{(N_S / t_S) - (N_B / t_B)}{\varepsilon}}{\partial \varepsilon} \right)^2 u^2(\varepsilon)$$

$$= \left(\frac{1/t_S}{\varepsilon} \right)^2 u^2(N_S) + \left(\frac{-1/t_B}{\varepsilon} \right)^2 u^2(N_B) + \left(\frac{-((N_S / t_S) - (N_B / t_B))}{\varepsilon^2} \right)^2 u^2(\varepsilon)$$

$$= \left(\frac{1/300}{0.4176} \right)^2 \sqrt{11578}^2 + \left(\frac{-1/6000}{0.4176} \right)^2 \sqrt{87}^2 + \left(\frac{-(11578/300) + (87/6000)}{0.4176^2} \right)^2 0.005802^2$$

= 0.73768 + 0.00001+ 1.64745 = 2.38515.

Note that these calculations show which input quantities are contributing the most to the combined variance. N_S contributes 0.73768/2.38515 ~ 31%. N_B contributes virtually nothing. The uncertainty in the efficiency contributes 1.64745/2.38515 ~ 69%. An analysis such as this is called an uncertainty budget, and quickly points out where improvements in the measurement may be made.

Taking the square root of the combined variance we find $u_c(y)$ = 1.54439. Usually the CSU is rounded to two significant figures and the result is rounded to match the same number of decimal places. So the result would be reported as 92.3 Bq with a CSU of 1.5 Bq.

Note that if the uncertainty in the efficiency had been neglected, the CSU would have been underestimated as 0.86 Bq, and would have been attributed entirely to the uncertainty in the sample counts. This illustrates the importance of including all significant sources of uncertainty in the calculations. Many of these calculations can be done using computer software programs mentioned earlier.

A much more detailed and involved example is given in Section 7.8.3.

Again, it should be noted that software (e.g., McCroan 2006, GUM Workbench 2006, Kragten 1994, Vetter 2006) is available to perform the partial derivatives, insert the proper mean and standard uncertainty for each input, and perform the algebra for uncertainty evaluation and propagation. This eliminates much of the tedium in implementing the uncertainty calculations, and frees the analyst to carefully examine the model equation to be sure that significant sources of uncertainty are not omitted.

7.4.3 Recommendations for Uncertainty Calculation and Reporting

- Use the terminology and methods of the GUM (ISO 1995) for evaluating and reporting measurement uncertainty.

- Follow QC procedures that ensure the measurement process remains in a state of statistical control, which is a prerequisite for uncertainty evaluation.
- Account for possible blunders or other spurious errors. Spurious errors indicate a loss of statistical control of the process and are not part of the uncertainty analysis described above.
- Report each measured value with either its CSU (or its expanded uncertainty, see Section 7.8.1.7).
- Reported measurement uncertainties should be clearly explained. (In particular, when an expanded uncertainty is reported, the coverage factor should be stated and the basis for the coverage probability should also be given, see Section 7.8.1.7).
- Consider all possible sources of measurement uncertainty and evaluate and propagate the uncertainties from all sources believed to be potentially significant in the final result.
- Each uncertainty should be rounded to either one or two significant figures and the measured value should be rounded to the same number of decimal places as its uncertainty.
- Results should be reported as obtained together with their uncertainties (whether positive, negative, or zero).

7.5 Determine Measurement Detectability

This section summarizes issues related to measurement detection capabilities. Much of this material is derived from MARLAP Chapter 20. More detail may be found in see Section 7.9.

Environmental radioactivity measurements may involve material with very small amounts of the radionuclide of interest. Measurement uncertainty often makes it difficult to distinguish such small amounts from zero. Therefore, an important MQO of a measurement process is its detection capability, which is usually expressed as the smallest concentration of radioactivity that can be reliably distinguished from zero. Effective project planning requires knowledge of the detection capabilities of the measurement method that will be or could be used. This section explains an MQO called the minimum detectable concentration (MDC) and describes radioactivity detection capabilities, as well as methods for calculating it.

The method most often used to make a detection decision about radiation or radioactivity involves the principles of statistical hypothesis testing. It is a specific example of a Scenario B hypothesis testing procedure described in Section 7.2.4. To "detect" the radiation or radioactivity requires a decision on the basis of the measurement data that the radioactivity is present. The detection decision involves a choice between the null hypothesis (H_0): There is no radiation or radioactivity present (above background), and the alternative hypothesis (H_1): There is radiation or radioactivity present (above background). In this context, a Type I error is to conclude that radiation or radioactivity is present when it actually is not, and a Type II error is to conclude that radiation or radioactivity is not present when it actually is.[5] Making the choice between these hypotheses requires the calculation of a critical value. If the measurement result exceeds this critical value, the null hypothesis is rejected and the decision is that radiation or radioactivity is present.

[5] Note that in any given situation only one of the two types of decision error is possible. If the sample *does not* contain radioactivity, a Type I error is possible. If the sample *does* contain radioactivity, a Type II error is possible.

7.5.1 Calculate the Critical Value

The critical value defines the lowest value of the net instrument signal[6] (count) that is too large to be compatible with the premise that there is no radioactivity present. It has become standard practice to make the detection decision by comparing the net instrument count to its critical value, S_C. The net count is calculated from the gross count by subtracting the estimated background and any interferences.[7]

The mean value of the net instrument count typically is positive when there is radioactivity present (i.e., above background). The gross count must be corrected by subtracting an estimate of the count produced under background conditions. See Section 7.8.2 for more information on instrument background.

Table 7.5 lists some formulas that are commonly used to calculate the critical value, S_C, together with the major assumptions made in deriving them. Note that the Stapleton formulas given in rows 3 through 5 especially are appropriate when the total background is less than 100 counts. These formulas depend on N_B (background count), t_B (background count time), t_S (sample count time), and $z_{1-\alpha}$ (the $(1-\alpha)$-quantile of the standard normal distribution). The value of α determines the sensitivity of the test. It is the probability that a detection decision is made when no radioactivity above background is actually present.

More detail on the calculation of critical values is given in Section 7.9.3. Software (Strom 1999) is available for calculating S_C using the equations recommended here, among others.

Table 7.5 Recommended Approaches for Calculating the Critical Value of the Net Instrument Signal (Count), S_C[8]

	Critical Value Equation	Assumptions	Background Count
1	$S_C = z_{1-\alpha}\sqrt{N_B \frac{t_S}{t_B}\left(1+\frac{t_S}{t_B}\right)}$	Poisson	> 100
2	$S_C = 2.33\sqrt{N_B}$	Poisson $\alpha = 0.05$ $t_B = t_S$	> 100

[6] "Net instrument signal," is used here as a general term, because many radiation-detection instruments may have output other than "counts" (e.g., current for ionization chambers). In cases where the instrument output *is* in counts, the term "net counts" can be substituted for the term "net instrument signal."

[7] "Interference" is the presence of other radiation or radioactivity or electronic signals that hinder the ability to analyze for the radiation or radioactivity of interest.

[8] These particular expressions for the critical value of the net instrument signal (in this case the net count) depend for their validity on the assumption of Poisson counting statistics. If the variance of the blank signal is affected by interferences, or background instability, then the Equation 20.7 of MARLAP may be more appropriate.

	Critical Value Equation	Assumptions	Background Count
3	$S_C = d \times \left(\frac{t_S}{t_B} - 1\right) + \frac{z_{1-\alpha}^2}{4} \times \left(1 + \frac{t_S}{t_B}\right) + z_{1-\alpha}\sqrt{\left(N_B + d\right)\frac{t_S}{t_B}\left(1 + \frac{t_S}{t_B}\right)}$	Stapleton $t_B \neq t_S$	< 100
4	$S_C = 0.4 \times \left(\frac{t_S}{t_B} - 1\right) + \frac{1.645^2}{4} \times \left(1 + \frac{t_S}{t_B}\right) + 1.645\sqrt{\left(N_B + 0.4\right)\frac{t_S}{t_B}\left(1 + \frac{t_S}{t_B}\right)}$	Stapleton $t_B \neq t_S$ $\alpha = 0.05$ $d = 0.4$	< 100
5	$S_C = 1.35 + 2.33\sqrt{N_B + 0.4}$	Stapleton $t_B = t_S$ $\alpha = 0.05$ $d = 0.4$	< 100

d = the critical value of the net instrument signal parameter in the Stapleton Equation

Example 7: A 600-second background measurement is performed on a proportional counter and 108 beta counts are observed. A sample is to be counted for 300 s. Estimate the critical value of the net instrument signal (i.e., net count) when $\alpha = 0.05$.

$$S_C = z_{1-\alpha}\sqrt{N_B \frac{t_S}{t_B}\left(1 + \frac{t_S}{t_B}\right)}$$

$$S_C = 1.645\sqrt{108 \times \left(\frac{300\text{ s}}{600\text{ s}}\right)\left(1 + \frac{300\text{ s}}{600\text{ s}}\right)} = 14.8 \text{ net counts}$$

Therefore, if 15 or more net counts are observed, the decision will be made that the sample contains radioactivity above background. Values of S_C should be rounded up when necessary to make sure that the specified Type I error probability, α, is not exceeded.

7.5.2 Calculate the Minimum Detectable Value of the Net Instrument Signal or Count

Table 7.6 lists some formulas that are commonly used to calculate the minimum detectable net count, S_D, together with the major assumptions made in deriving them. S_D, is defined as the mean value of the net instrument signal or count that gives a specified probability, $1 - \beta$, of yielding an observed net instrument signal or count greater than its critical value S_C. Therefore, S_C must be calculated before S_D. Note specifically that the Stapleton formulas given in rows 4 and 5 are especially appropriate when the total background is less than 100 counts. Generally, the Stapleton methods may be used for both high and low total background counts as they agree well with the more traditional methods when the background counts are over 100. The simpler, more familiar formulas have been included for completeness.

It is important that the assumptions used to calculate S_D are consistent with those that were used to calculate S_C. The equations for S_D depend on the same variables as S_C, namely N_B, t_B, and t_S. Notice that neither α nor $z_{1-\alpha}$ appears explicitly, rather they enter the calculation through S_C. However, β now enters the calculation of S_D through $Z_{1-\beta}$. The value of β, like α, is usually chosen to be 0.05 or is assumed to be 0.05 by default if no value is specified.

Table 7.6 Recommended Approaches for Calculating the Minimum Detectable Net Instrument Signal or Count[9]

	Minimum Detectable Net Signal Equation	Assumptions	Background Count
1	$S_D = S_C + \frac{z_{1-\beta}^2}{2} + z_{1-\beta}\sqrt{\frac{z_{1-\beta}^2}{4} + S_C + N_B \frac{t_S}{t_B}\left(1+\frac{t_S}{t_B}\right)}$	Poisson $t_B \neq t_S$	> 100
2	$S_D = z_{1-\beta}^2 + 2S_C$	Poisson $t_B \neq t_S$ $\alpha = \beta$	> 100
3	$S_D = 2.71 + 2S_C = 2.71 + 2(2.33\sqrt{N_B}) = 2.71 + 4.66\sqrt{N_B}$	Poisson $\alpha = \beta = 0.05$ $t_B = t_S$	> 100
4	$S_D = \frac{(z_{1-\alpha} + z_{1-\beta})^2}{4}\left(1+\frac{t_S}{t_B}\right) + (z_{1-\alpha} + z_{1-\beta})\sqrt{N_B \frac{t_S}{t_B}\left(1+\frac{t_S}{t_B}\right)}$	Stapleton	< 100
5	$S_D = 5.41 + 4.65\sqrt{N_B}$	Stapleton $\alpha = \beta = 0.05$ $t_B = t_S$	< 100

Example 8 A 600-second background measurement on a proportional counter produces 108 beta counts and a source is to be counted for 300 s. Assume the background measurement gives the available estimate of the true mean background count rate and use the value 0.05 for Type I and Type II error probabilities. From section 7.5.1, Example 7, the critical net count, S_C, equals 14.8, so $S_D = z_{1-\beta}^2 + 2S_C = 1.645^2 + 2\,(14.8) = 32.3$ net counts. Values of S_D should be rounded up when necessary to make sure that the specified Type II error probability, β, is not exceeded.

The relationship between the critical value of the net instrument signal (or count), S_C, and the minimum detectable net instrument signal (or count), S_D, is shown in Figure 7.6. Figure 7.6 illustrates a case where alpha is greater than beta. The net instrument signal (or count) obtained for a blank sample will usually be distributed around zero as shown. Occasionally, a net count rate above S_C may be obtained by chance. The probability that this happens is controlled by the value of α, shown as the lightly shaded area in Figure 7.6. Smaller values of α result in larger values of S_C and vice versa. The minimum detectable value of the net instrument signal (or count) S_D is that value of the mean net instrument signal (or count) that results in a detection decision with probability $1-\beta$. That is, there is only a probability equal to β, shown as the more

[9] These expressions for the critical value of the net count depend for their validity on the assumption of Poisson counting statistics. If the variance of the blank signal is affected by interferences, or background instability, then Equation 20.7 of MARLAP may be more appropriate. "Interference" is the presence of other radiation or radioactivity or electronic signals that hinder the ability to analyze for the radiation or radioactivity of interest.

darkly shaded area in Figure 7.6, of yielding an observed count less than S_C. Smaller values of β result in larger values of S_D and vice versa.

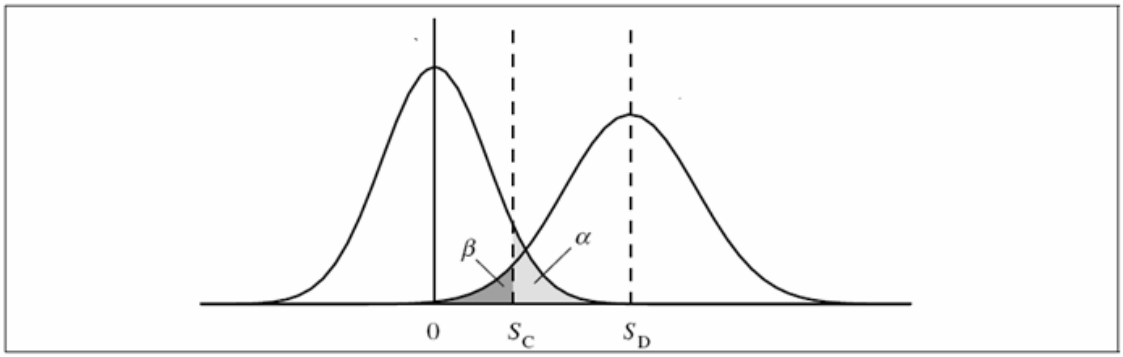

Figure 7.6 The Critical Value of the Net Instrument Signal (S_C) and the Minimum Detectable Net Signal (S_D)

More information detail on the calculation of the minimum detectable value of the net instrument signal, S_D, is given in Section 7.9.

7.5.3 Calculate the Minimum Detectable Concentration

The MDC is usually obtained from the minimum detectable value of the net instrument signal (or count), S_D. The MDC is by definition an estimate of the true concentration of the radiation or radioactivity required to give a specified high probability that the measured response will be greater than the critical value. The common practice of comparing a measured concentration to the MDC, instead of to the S_C, to make a detection decision is incorrect. To calculate the MDC, the minimum detectable value of the net signal (or count), S_D, must first be converted to the detectable value of the net instrument signal per unit time (or count rate), $S_D / t_S(s^{-1})$. This in turn must be divided by the counting efficiency, ε (s^{-1})/Bq to get the minimum detectable activity, y_D. Finally, the minimum detectable activity can be divided by the sample volume or mass to obtain the MDC. At each stage in this process, additional uncertainty may be introduced by the uncertainties in time, efficiency, volume, mass, etc. Thus, prudently conservative values of these factors should be used so that the desired detection power, $1-\beta$, at the MDC is maintained. Another approach would be to recognize that y_D itself has an uncertainty which can be calculated using the methods of Section 7.4. Thus any input quantity that is used to convert from S_D to y_D that has significant uncertainty can be incorporated to assess the overall uncertainty in the MDC. Additional discussion of the calculation of the MDCs is given in Section 7.9.5.

Example 9: Continuing Example 8, $S_D = 32.3$ net counts.

Assuming negligible uncertainty in the count time, the net count rate is $S_D / t_S = 32.3/300 = 0.1077\ (s^{-1})$.

The mean efficiency from Example 6 in Section 7.4.2 was 0.4176 (s^{-1})/(Bq) with a CSU of $u_C(\varepsilon) = 0.005802$.

In Example 8, the value 0.05 was specified for both Type I and Type II error probabilities. So the specified power was $1-\beta = 1 - 0.05 = 0.95$.
Assume a normal distribution for ε, to obtain a 95% probability of detection for the MDC. To account for the variability in the efficiency, the value used for ε should be the 5th percentile, i.e., 0.4176 – 1.645(0.005802) = 0.4081.

Thus, the minimum detectable activity, $y_D = \frac{S_D / t_s}{\varepsilon} = 0.1077 / 0.4081 = 0.2639$ Bq.
Using the mean value of the efficiency would potentially underestimate the minimum detectable activity as $y_D = \frac{S_D / t_s}{\varepsilon} = 0.1077 / 0.4176 = 0.2578$ Bq.
These values for y_D would then be divided by the mass or volume of the sample to yield the MDC.

7.5.4 Summary of Measurement Detectability

The concepts surrounding the MDC and the critical value are illustrated in Figure 7.7, using familiar formulae for S_C and S_D discussed above, assuming a background count of $N_B = 100$ with $\alpha = \beta = 0.5$. In this case, the equation in row 2 of Table 7.5 was used to obtain $S_C = 23.3$, and the corresponding equation in row 3 of Table 7.6 to obtain $S_D = 49.3$. The use of these equations implies $\alpha = \beta = 0.05$ and $t_B = t_S$. It is important to note that traditionally the values $\alpha = \beta = 0.05$ are used for MDC calculations, so that the MDCs for different methods are comparable. However, when developing a standard operating procedure for a survey, other values for α and β may be more appropriate. A case where this typically occurs is in the calculation of scan MDCs (Section 7.11.6) where α may be much greater than β, because the consequences associated with misidentifying a background area as elevated are much lower than the consequences associated with missing a true elevated area.

Note, the upper abscissa scale is in concentration and the lower abscissa scale is in net count. These are related by the efficiency at the point where the MDC corresponds to the minimum detectable net instrument signal (or count), S_D. Each of the curves illustrates the distribution of mean net counts (or concentration) that may exist for a measurement. The width of these curves represents the variation due to counting statistics. The variability due to other factors is associated with uncertainty in ε. Changes in the relationship between the lower and the upper scales result from changes in ε. This illustrates the importance of choosing realistic, or even conservative, values of ε. Note that the probability of making a detection decision (which is proportional to the area of each curve to the right of S_C) depends on the concentration, increasing from 5% at background to 95% at the MDC, passing through 50% at S_C. This is perhaps more clearly shown in Figure 7.8, which plots the probability of making a detection decision as a function of net instrument signal, count, or concentration.

Figure 7.8 shows that for concentrations corresponding to net counts between 0 and S_C the probability of a non-detect is greater than 50%. For concentrations corresponding to net counts between S_C and S_D the probability of detection is greater than 50%, but less than 95%.

Concentrations above the MDC (with net counts greater than S_D) are highly likely to be detected, but will have relative standard uncertainties that are somewhat large.

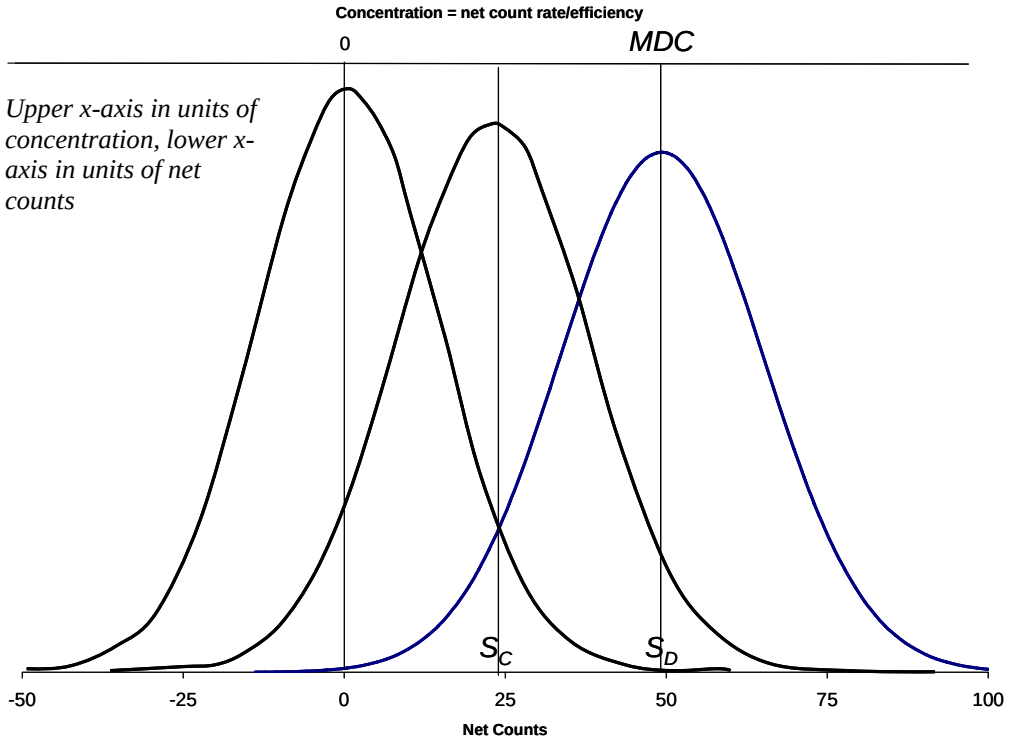

Figure 7.7 Relationship Between the Critical Value of the Net Count, the Minimum Detectable Net Counts and the MDC

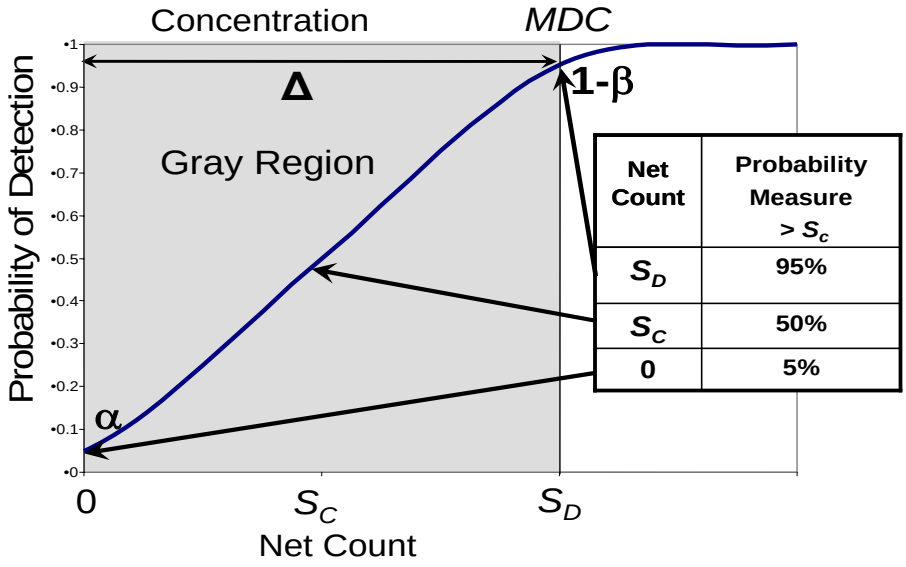

Figure 7.8 Probability of Detection as a Function of Net Count (Lower X-Axis) and Concentration (Upper X-Axis)

7.5.5 Measurement Detectability Recommendations

- When a detection decision is required, generally it should be made by comparing the net instrument signal (or count) to its corresponding critical value.
- Expressions from Tables 7.5 and 7.6 for S_C and S_D should be chosen to match the assumptions and background for the measurement method.
- An appropriate background should be used to predict the instrument signal produced when there is no radioactivity present in the sample.
- The Minimum Detectable Concentration (MDC) should be used only as a MQO for the measurement method. To make a detection decision, a measurement result should be compared the critical value and never to the MDC.
- The validity of the Poisson approximation for the measurement process should be confirmed using the methods described in MARLAP Chapter 20 before using an expression for the critical value that is based on Poisson statistics. When the Poisson approximation is inappropriate for determining the critical value, estimating σ by the sample standard deviation of replicated background measurements is preferable to using the square root of the number of counts.
- Consider all significant sources of variance in the instrument signal (or other response variable) when calculating the critical value, S_C, and minimum detectable value, S_D.
- Report each measurement result and its uncertainty as obtained even if the result is less than zero. Never report a result as "less than MDC" or "less than S_C."
- The MDC should not be used for projects where the issue is a quantitative comparison of measurements to a limit rather than just a detection decision made for a single measurement. For these projects, the minimum quantifiable concentration is a more relevant MQO for the measurement process (see Section 7.6).

7.6 Determine Measurement Quantifiability

This section discusses issues related to measurement quantifiability. Much of this material is derived from the MARLAP Chapter 20. Further details and an additional example are given in Section 7.10.

Action levels are frequently stated in terms of a quantity or concentration of radioactivity, rather than in terms of detection. In these cases, project planners may need to know the quantification capability of a measurement method, or its capability for precise measurement. The quantification capability is expressed as the smallest concentration of radiation or radioactivity that can be measured with a specified relative standard deviation. This section explains an MQO called the minimum quantifiable concentration (MQC), which may be used to describe quantification capabilities.

The MQC of the concentration, y_Q, is defined as the concentration at which the measurement process gives results with a specified relative standard deviation $1/k_Q$ where k_Q is usually chosen to be 10 for comparability.

Historically much attention has been given to the detection capabilities of radiation and radioactivity measurement processes, but less attention has been given to quantification capabilities. For some projects, quantification capability may be a more relevant issue. For example, suppose the purpose of a project is to determine whether the ^{226}Ra concentration on material at a site is below an action level. Since ^{226}Ra can be found in almost any type of naturally occurring material, it may be assumed to be present in every sample, making detection decisions unnecessary. The MDC of the measurement process obviously should be less than the action level, but a more important question is whether the MQC is less than the action level.

A common practice in the past has been to select a measurement method based on the minimum detectable concentration (MDC), which is defined in Section 7.5. For example, MARSSIM (2002) says:

> *During survey design, it is generally considered good practice to select a measurement system with an MDC between 10-50% of the DCGL [action level].*

Such guidance implicitly recognizes that for cases when the decision to be made concerns the mean of a population that is represented by multiple measurements, criteria based on the MDC may not be sufficient and a somewhat more stringent requirement is needed. The requirement that the MDC (approximately 3-5 times σ_M) be 10% to 50% of the action level is tantamount to requiring that σ_M be 0.02 to 0.17 times the action level – in other words, the relative standard deviation should be approximately 10% at the action level. However, the concentration at which the relative standard deviation is 10% is the MQC when k_Q assumes its conventional value of 10. Thus, a requirement that is often stated in terms of the MDC may be more naturally expressed in terms of the MQC, e.g., by saying that the MQC should not exceed the action level.

7.6.1 Calculate the MQC

The minimum quantifiable concentration, when there are no interferences, can be calculated from:

$$y_Q = \frac{k_Q^2}{2t_S\varepsilon(1-k_Q^2\phi_{\hat{\varepsilon}}^2)}\left(1+\sqrt{1+\frac{4(1-k_Q^2\phi_{\hat{\varepsilon}}^2)}{k_Q^2}\left(N_B\frac{t_S}{t_B}\left(1+\frac{t_S}{t_B}\right)\right)}\right) \quad (7\text{-}14)$$

Where:

t_S = count time for the source, s
t_B = count time for the background, s
N_B = background count
$\phi_{\hat{\varepsilon}}^2$ = relative variance of the measured efficiency, $\hat{\varepsilon}$ (see Section 7.8.2.2)
k_Q = relative percent standard deviation at the MQC, usually assumes a conventional value of 10 for purposes of comparison among methods

If $k_Q^2\,\phi_{\hat{\varepsilon}}^2 \geq 1$, this equation has no solution.

Example 10: Continuing Example 9, $t_S = 300$, $t_B = 600$, $N_B = 108$, $\phi_{\hat{\varepsilon}}^2 = (0.005802/0.4176)^2 = 0.0001932$, and $k_Q = 10$. So,

$$y_Q = \frac{k_Q^2}{2t_S\varepsilon(1-k_Q^2\phi_{\hat{\varepsilon}}^2)}\left(1+\sqrt{1+\frac{4(1-k_Q^2\phi_{\hat{\varepsilon}}^2)}{k_Q^2}\left(N_B\frac{t_S}{t_B}\left(1+\frac{t_S}{t_B}\right)\right)}\right)$$

$$= \frac{100}{2(300)(0.4176)(1-100(0.0001932))}\left(1+\sqrt{1+\frac{4(1-100(0.0001932))}{100}\left(108\frac{300}{600}\left(1+\frac{300}{600}\right)\right)}\right)$$

= 1.239 Bq. This value for y_Q would then be divided by the mass or volume of the sample to yield the MQC.

The next example is given to verify that the equation for y_Q does indeed produce a value with a relative uncertainty of 10%. It also provides an opportunity to give another illustration of the methodology for the calculation of measurement uncertainty developed in Sections 7.4 and 7.8. Additional information on the calculation of MQCs is given in Section 7.10.

Example 11: The calculations of Example 10 can be verified by calculating the uncertainty of a measurement made at the MQC. The expected number of counts for a sample at the MQC counted for 300 s:

$$N_S = y_Q t_S \varepsilon + N_B(t_S/t_B) = (1.239\text{ Bq})(300\text{ s})(0.4176) + (108\text{ s}^{-1})(300/600) = 209,$$

rounded to the nearest whole number.

The model equation is the same as was used in Example 6, Section 7.4.2:

$y = \frac{(N_S/t_S)-(N_B/t_B)}{\varepsilon}$, so the equation for the CSU is the same:

$$u_c^2(y) = \left(\frac{1/t_S}{\varepsilon}\right)^2 u^2(N_S) + \left(\frac{-1/t_B}{\varepsilon}\right)^2 u^2(N_B) + \left(\frac{-((N_S/t_S)-(N_B/t_B))}{\varepsilon^2}\right)^2 u^2(\varepsilon)$$

$$= \left(\frac{1/300}{0.4176}\right)^2 (209) + \left(\frac{-1/600}{0.4176}\right)^2 (108) + \left(\frac{-(209/300)+(108/600)}{0.4176^2}\right)^2 (0.005802)^2$$

$$= 1.332\times10^{-2} + 1.72\times10^{-3} + 2.95\times10^{-4} = 1.534\times10^{-2}$$

$u_c(y) = \sqrt{1.534\times10^{-2}} = 0.124$. Thus, the relative uncertainty at the MQC is 0.124/1.239 = 0.09995. This means, apart from some small difference due to rounding, the relative measurement uncertainty at y_Q is 10%, as should be the case for the MQC.

7.6.2 Summary of Measurement Quantifiability

Figure 7.9 is a modification of Figure 7.8, illustrating the relationships between the critical value, the MDC, the MQC and the probability of exceeding the critical value. As can be seen, the issue of detection is almost moot at the MQC. The probability of detection is near 100%. However, the

MQC specifies a concentration with a defined relative standard uncertainty, making comparisons between measurements or comparisons between measurements and regulatory criteria meaningful.

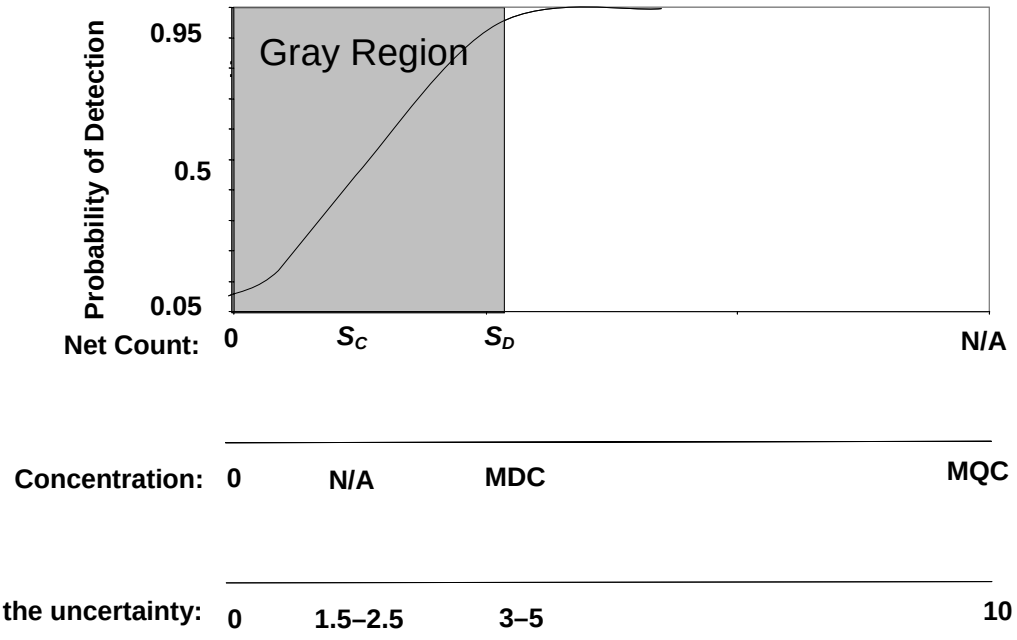

Figure 7.9 Relationships Among the Critical Value, the MDC, the MQC, and the Probability of Exceeding the Critical Value

Three x-axis scales are shown in Figure 7.9 for net count, concentration, and multiple of measurement uncertainty. This figure emphasizes, for example, that the minimum detectable net count, S_D, corresponds to the MDC, but has different units. It also shows that the MQC is by convention 10 times the measurement uncertainty at that concentration. The critical value of the net count, S_C, has no corresponding common term in concentration units. This is because detection decisions are usually made on the basis of the net counts (instrument reading). These are inherently qualitative "yes or no" decisions. The relationship between S_C and S_D and the multiple of the uncertainty varies according to which set of assumptions are used and which equations in Table 7.5 and Table 7.6 are appropriate to those assumptions. Therefore, an approximate range is shown for these quantities on the multiple of uncertainty axis.

7.7 Establish a Required Measurement Method Uncertainty

This section provides the rationale and guidance for establishing project-specific MQOs for controlling σ_M and expands on the material in Section 7.3. Control of σ_M is achieved by establishing a desired maximum measurement method uncertainty at the upper boundary of the gray region. This control also will assist in both the measurement method selection process and in the evaluation of measurement data. Approaches applicable to several situations are detailed below.

7.7.1 Developing a Requirement for Measurement Method Uncertainty for MARSSIM-Type Surveys

When, as in MARSSIM-type surveys, a decision is to be made about the mean of a sampled population, generally the average of a set of measurements on a survey unit is compared to the disposition criterion.

The total variance of the data, σ^2, is the sum of two components

$$\sigma^2 = \sigma_M^2 + \sigma_S^2 \tag{7-15}$$

Where:

σ_M^2 = measurement method variance (M for measurement)

σ_S^2 = variance of the radionuclide concentration or activity concentration in the sampled population (S for sampling)

The spatial and temporal distribution of the concentration (i.e., the variation of the true but unknown concentrations from place to place and from time to time), the extent of the survey unit, the physical sizes of the measured material, and the choice of measurement locations may affect the sampling standard deviation, σ_S. The measurement standard deviation, σ_M, is affected by the measurement methods. The value of σ_M is estimated in MARSAME by the CSU of a measured value for a measurement of material whose concentration equals the hypothesized population mean concentration. The calculation of measurement uncertainties is covered in Sections 7.4 and 7.8.

Four cases are considered below where target values for σ_M can be suggested depending on what is known about σ_S. Cases 1 and 2 treat the desired overall objective of keeping $\Delta/\sigma \approx 3$ or higher. When this is not possible, Cases 3 and 4 treat the less desirable alternative of attempting to prevent Δ/σ from going lower than 1. If $\Delta/\sigma < 1$ then a large number of measurements will be required to meet the Type I and II decision error rates specified in the DQO process. If $\sigma \gg \Delta$, it may be necessary to re-evaluate the error rates specified in the DQO process.

Case 1: σ_S is known relative to $\Delta / 3$

Generally, it is easier to control σ_M than σ_S. If σ_S is known (approximately), a target value for σ_M can be determined.

Case 1a: $\sigma_S \leq \Delta/3$

If $\sigma_S \leq \Delta/3$, then a value of σ_M no greater than $\sqrt{(\Delta^2/9) - \sigma_S^2}$ ensures that $\sigma \leq \Delta / 3$, because we have $\sigma^2 = \sigma_M^2 + \sigma_S^2 \leq (\Delta^2/9 - \sigma_S^2) + \sigma_S^2 = \Delta^2/9$, as desired.

Case 1b: $\sigma_S > \Delta/3$

If $\sigma_S > \Delta/3$, the requirement that the total σ be less than $\Delta/3$ cannot be met regardless of σ_M. In this case, it is sufficient to make σ_M negligible in comparison to σ_S. Generally, σ_M can be considered negligible in comparison to σ_S if $\sigma_M < \sigma_S/3$.

Case 2: σ_S is not known relative to $\Delta/3$

Often one needs a method for choosing σ_M in the absence of specific information about σ_S. Since it is desirable to have $\sigma \leq \Delta/3$, this condition is adopted as a primary requirement. Assume for the moment that σ_S is large. Then σ_M should be made negligible by comparison. As mentioned above, σ_M can be considered negligible if it is no greater than $\sigma_S/3$. When this condition is met, further reduction of σ_M has little effect on σ and therefore is usually not cost-effective. So, $\sigma_M \leq \sigma_S/3$ is adopted as a secondary requirement.

Starting with the definition $\sigma^2 = \sigma_M^2 + \sigma_S^2$ and substituting the secondary requirement $\sigma_M \leq \sigma_S/3$ we get $\sigma^2 \geq \sigma_M^2 + 9\sigma_M^2 = 10\sigma_M^2$, thus

$$\sigma_M \leq \frac{\sigma}{\sqrt{10}} \tag{7-16}$$

Substituting the primary requirement that $\Delta/\sigma \geq 3$ (i.e., $\sigma \leq \Delta/3$) we get $\sigma_M \leq \frac{\sigma}{\sqrt{10}} \leq \frac{\Delta/3}{\sqrt{10}}$, thus

$$\sigma_M \leq \frac{\Delta}{3\sqrt{10}} \tag{7-17}$$

Or approximately

$$\sigma_M \leq \frac{\Delta}{10} \tag{7-18}$$

The required upper bound for the standard deviation σ_M will be denoted by σ_{MR}. MARSAME recommends the equation

$$\sigma_{MR} = \frac{\Delta}{10} \tag{7-19}$$

by default as a requirement when σ_S is unknown and a decision is to be made about the mean of a sampled population.

This upper bound was derived from the assumption that σ_S was large, but it also ensures that the primary requirement $\sigma \leq \Delta/3$ (i.e., $\Delta/\sigma \geq 3$) will be met if σ_S is small. When the measurement standard deviation σ_M is less than σ_{MR}, the primary requirement will be met unless the sampling variance, σ_S^2, is so large that σ_M^2 is negligible by comparison, in which case little benefit can be obtained from further reduction of σ_M.

It may be that the primary requirement that Δ/σ be at least 3 is not achievable. Suppose that the primary requirement is relaxed to achieving Δ/σ at least 1 (i.e., $\sigma \leq \Delta$). This leads to consideration of–

Case 3: σ_S is known relative to Δ

As in Case 1, it is generally easier to control σ_M than σ_S. If σ_S is known (approximately), a target value for σ_M can be determined.

Case 3a: $\sigma_S \leq \Delta$
If $\sigma_S \leq \Delta$, then a value of σ_M no greater than $\sqrt{\Delta^2 - \sigma_S^2}$ ensures that $\sigma \leq \Delta$, because we have $\sigma^2 = \sigma_M^2 + \sigma_S^2 \leq (\Delta^2 - \sigma_S^2) + \sigma_S^2 = \Delta^2$ as desired.

Case 3b: $\sigma_S > \Delta$
If $\sigma_S > \Delta$, the requirement that the total σ be less than Δ cannot be met regardless of σ_M. In this case, it is sufficient to make σ_M negligible in comparison to σ_S. Generally, σ_M can be considered negligible if it $\sigma_M < \sigma_S/3$.

Case 4: σ_S is not known relative to Δ

Suppose $\sigma \leq \Delta$ is adopted as the primary requirement. As in Case 2, if σ_S is large, then σ_M should be made negligible by comparison. As mentioned above, σ_M can be considered negligible if it is no greater than $\sigma_S/3$. When this condition is met, further reduction of σ_M has little effect on σ and therefore is usually not cost-effective. So, $\sigma_M \leq \sigma_S/3$ is adopted as a secondary requirement.

Starting with the definition $\sigma^2 = \sigma_M^2 + \sigma_S^2$ and substituting the secondary requirement $\sigma_M \leq \sigma_S/3$ we get $\sigma^2 \geq \sigma_M^2 + 9\sigma_M^2 = 10\sigma_M^2$, thus–

$$\sigma_M \leq \frac{\sigma}{\sqrt{10}} \tag{7-20}$$

Substituting the primary requirement that $\Delta/\sigma \geq 1$ (i.e., $\sigma \leq \Delta$) we get $\sigma_M \leq \frac{\sigma}{\sqrt{10}} \leq \frac{\Delta}{\sqrt{10}}$, thus–

$$\sigma_M \leq \frac{\Delta}{\sqrt{10}} \approx \frac{\Delta}{3} \tag{7-21}$$

7.7.2 Developing a Requirement for Measurement Method Uncertainty When Decisions are to be Made About Individual Items

When decisions are to be made about individual items, the total variance of the data equals the measurement variance, σ_M^2, and the data distribution in most instances should be approximately normal. The decision in this case may be made by comparing the measured concentration, *x*, plus or minus a multiple of its CSU, to the action level. The CSU, $u_c(x)$, is assumed to be an estimate of the true standard deviation of the measurement process as applied to the item being measured;

so, the multiplier of $u_c(x)$ equals $z_{1-\alpha}$, the $(1-\alpha)$-quantile of the standard normal distribution (see MARLAP appendix C).

Alternatively, if AL = 0, so that any detectable amount of radioactivity is of concern, the decision may involve comparing the net instrument signal (e.g., count rate) to the critical value of the net instrument signal, S_C, as defined in Section 7.5.1.

Two cases are considered below where target values for σ_M can be suggested depending on what is known about the width of the gray region and the desired Type I and Type II decision error rates. Case 5 is for Scenario A, and Case 6 is for Scenario B.

Case 5: Suppose the null hypothesis is $X \geq$ AL (see Scenario A in Chapter 4), so that the action level is the upper bound of the gray region. Given the measurement variance σ_M^2, only a measured result that is less than (UBGR – $z_{1-\alpha}\,\sigma_M$) will be judged to be clearly less than the action level. Then the desired power of the test $1-\beta$ is achieved at the lower bound of the gray region only if the LBGR $\leq$ UBGR – $z_{1-\alpha}\sigma_M - z_{1-\beta}\sigma_M$. Algebraic manipulation transforms this requirement to

$$\sigma_M \leq \frac{\text{UBGR - LBGR}}{z_{1-\alpha} + z_{1-\beta}} = \frac{\Delta}{z_{1-\alpha} + z_{1-\beta}} \tag{7-22}$$

Case 6: Suppose the null hypothesis is $X \leq$ AL (see Scenario B in Chapter 4), so that the action level is the lower bound of the gray region. In this case, only a measured result that is greater than LBGR + $z_{1-\alpha}\sigma_M$ will be judged to be clearly greater than the action level. The desired power of the test $1 - \beta$ is achieved at the upper bound of the gray region only if the UBGR $\geq$ LBGR + $z_{1-\alpha}\sigma_M + z_{1-\beta}\sigma_M$. Algebraic manipulation transforms this requirement to:

$$\sigma_M \leq \frac{\text{UBGR - LBGR}}{z_{1-\alpha} + z_{1-\beta}} = \frac{\Delta}{z_{1-\alpha} + z_{1-\beta}} \tag{7-23}$$

So, in either Scenario A or Scenario B, the requirement remains that:

$$\sigma_M \leq \frac{\Delta}{z_{1-\alpha} + z_{1-\beta}} \tag{7-24}$$

Therefore, MARSAME uses the equation:

$$u_{MR} = \sigma_{MR} = \frac{\Delta}{z_{1-\alpha} + z_{1-\beta}} \tag{7-25}$$

as an MQO for method uncertainty when decisions are to be made about individual items or locations and not about population parameters.

If $\alpha = \beta = 0.05$, one may use the value $u_{MR} = 0.3\Delta$. Other combinations of α and β may lead to a similar result, but the relationship is nonlinear (depending on the standard normal distribution function) so one cannot simply apply a proportionality factor. Equation 7-25 must be used,

The recommended value of u_{MR} is based on the assumption that any known bias in the measurement process has been corrected and that any remaining bias is well less than a third of the method uncertainty.

7.8 Calculate the Combined Standard Uncertainty of a Measurement

This section expands upon the material in Section 7.4. Calculations of combined standard uncertainties (CSUs) can be complex, and typically would be carried out using a software package. For the purpose of illustration and clarity, fully worked out examples are included in this section.

7.8.1 Procedures for Evaluating Uncertainty

The usual eight steps for evaluating and reporting the uncertainty of a measurement are summarized in the following subsections (adapted from Chapter 8 of GUM):

7.8.1.1 Identify the Measurand, *Y*, and all the Input Quantities, X_i, for the Mathematical Model

Include all quantities whose variability or uncertainty could have a potentially significant effect on the result. Express the mathematical relationship, $Y = f(X_1, X_2,\ldots,X_N)$, between the measurand and the input quantities.

The procedure for assessing the uncertainty of a measurement begins with listing all significant sources of uncertainty in the measurement process. A good place to begin is with the input quantities' mathematical model $Y = f(X_1, X_2,\ldots,X_N)$. When an effect in the measurement process that is not explicitly represented by an input quantity has been identified and quantified, an additional quantity should be included in the mathematical measurement model to correct for it. The quantity, called a correction (additive with a nominal value of zero) or correction factor (multiplicative with a nominal value of one), will have an uncertainty that should also be evaluated and propagated. Each uncertainty that is potentially significant should be evaluated quantitatively.

7.8.1.2 Determine an Estimate, x_i, of the Value of Each Input Quantity, X_i

This involves simply determining for the particular measurement at hand, the specific value, x_i, that should be substituted for the input quantity X_i in the mathematical relationship, $Y = f(X_1, X_2,\ldots,X_N)$.

7.8.1.3 Evaluate the Standard Uncertainty, $u(x_i)$, for Each Input Estimate, x_i, Using a Type A Method, a Type B Method, or a Combination of Both

Methods for evaluating standard uncertainties are classified as either "Type A" or "Type B" (NIST 1994). Both types of uncertainty need to be taken into consideration. A Type A evaluation of an uncertainty uses a series of measurements to estimate the standard deviation empirically. Any other method of evaluating an uncertainty is a Type B method. A Type B evaluation of standard uncertainty is usually based on scientific judgment using all the relevant information available, which may include:

- Previous measurement data,
- Experience with, or general knowledge of, the behavior and property of relevant materials and instruments,
- Manufacturer's specifications,
- Data provided in calibration and other reports, and
- Uncertainties assigned to reference data taken from handbooks.

The Type A standard uncertainty of the input estimate x_i is defined to be the experimental standard deviation of the mean:

$$u(x_i) = \sqrt{\frac{1}{n(n-1)}\sum_{k=1}^{n}(x_{i,k} - \overline{x_i})^2} = s(x_i)/\sqrt{n} \qquad (7\text{-}26)$$

Example 12: Type A uncertainty calculation using Equation 7-26**:**

Ten independent one-minute measurements of the counts from a check source X_i were made with a digital survey meter, yielding the values:

12,148, 12,067, 12,207, 12,232, 12,284, 12,129, 11,862, 11,955, 12,044, and 12,150.

The estimated value x_i is the arithmetic mean of the values $X_{i,k}$.

$$x_i = X_i \frac{1}{n}\sum_{k=1}^{n} x_{i,k} = \frac{121078}{10} = 12107.8$$

The standard uncertainty of x_i is

$$u(x_i) = \sqrt{\frac{1}{n(n-1)}\sum_{k=1}^{n}(x_{i,k} - \overline{x_i})^2} = \sqrt{\frac{1}{10(10-1)}\sum_{k=1}^{10}(x_{i,k} - 12107.8)^2}$$

$$= \sqrt{16628.84} = 128.95$$

There are other Type A methods, but all are based on repeated measurements.

Any evaluation of standard uncertainty that is not a Type A evaluation is a Type B evaluation. Sometimes a Type B evaluation of uncertainty involves making a best guess based on all available information and professional judgment. Despite the reluctance to make this kind of evaluation, it is almost always better to make an informed guess about an uncertainty component than to ignore it completely.

There are many ways to perform Type B evaluations of standard uncertainty. One example of a Type B method is the estimation of counting uncertainty using the square root of the observed counts. If the observed count is N, when the Poisson approximation is used, the standard uncertainty of N may be evaluated as $u(N) = \sqrt{N}$.

Example 13: The standard uncertainty of the first value in Example 12, (12,148 counts), could be estimated as $\sqrt{12148} = 110.218$ counts. When *N* may be very small or even zero, the equation $u(N) = \sqrt{N+1}$ may be preferable.

Another Type B evaluation of an uncertainty $u(x)$ consists of estimating an upper bound, *a*, for the magnitude of the error of *x* based on professional judgment and the best available information. If nothing else is known about the distribution of the measured result, then after *a* is estimated, the standard uncertainty may be calculated using the equation

$$u(x) = \frac{a}{\sqrt{3}} \tag{7-27}$$

which is the standard deviation of a random variable uniformly distributed over the interval $(x - a, x + a)$. The variable *a* is called the half-width of the interval.

Example 14: Suppose in Example 12, all that was given was the observed range of the data from an analog survey meter dial (i.e., from 11,862 to 12,284), a difference of 422. If it was assumed that the data came from a uniform distribution across this range, then the average is

(11,862+12,284)/2 = 12,073

the half-width is 211, and an estimate of the standard uncertainty would be

$$u(x) = \frac{211}{\sqrt{3}} = 121.821$$

Given the same information on the range, if values near the middle of the range were considered more likely than those near the endpoint, a triangular distribution may be more appropriate. The standard uncertainty for a triangular distribution is calculated using the equation

$$u(x) = \frac{a}{\sqrt{6}} \tag{7-28}$$

which represents the standard deviation of a random variable with a triangular distribution over the interval $(x - a, x + a)$. Given the same information on the range, if values near the middle of the range were considered more likely than those near the endpoints, a triangular distribution may be more appropriate. The mean would be the same as above, 12,073. However the standard uncertainty then be calculated using the equation

$$u(x) = \frac{a}{\sqrt{6}} = \frac{211}{\sqrt{6}} = 86.14 \tag{7-29}$$

Example 15: As in Example 14, all that is given was the observed range of the data from an analog survey meter dial, i.e., from 11,862 to 12,284, a difference of 422. If it was assumed that the data came from a triangular distribution across this range, then the average is (11,862+12,284)/2 = 12,073, the half-width is 211, and an estimate of the standard uncertainty would be

$$u(x) = \frac{a}{\sqrt{6}} = \frac{211}{\sqrt{6}} = 86.14$$

When the estimate of an input quantity is taken from an external source, such as a book or a calibration certificate, the stated standard uncertainty can be used.

7.8.1.4 Evaluate the Covariances, $u(x_i,x_j)$, for all Pairs of Input Estimates with Potentially Significant Correlations

A Type A evaluation of the covariance of the input estimates x_i = and x_j = is

$$u(x_i, x_j) = \frac{1}{n(n-1)} \sum_{k=1}^{n} (x_{i,k} - \overline{x_i})(x_{j,k} - \overline{x_j}) \quad (7\text{-}30)$$

An evaluation of variances and covariances of quantities determined by the method of least squares may also be a Type A evaluation. Evaluation of the covariance of two input estimates, x_i and x_j, whose uncertainties are evaluated by Type B methods may require expert judgment. In such cases it may be simpler to estimate the correlation coefficient,

$$r(x_i,x_j) = [u(x_i,x_j) / u(x_i) \cdot u(x_j)] \quad (7\text{-}31)$$

first and then multiply it by the standard uncertainties, $u(x_i)$ and $u(x_j)$ to obtain the covariance, $u(x_i,x_j)$.

A covariance calculation is demonstrated in Example 16 in Section 7.8.2.2.

7.8.1.5 Calculate the Estimate, y, of the Measurand from the Relationship $y = f(x_1,x_2,\ldots,x_N)$

This involves simply substituting, for the particular measurement at hand, the specific values of x_i for the input quantity X_i into the mathematical relationship, $Y = f(X_1,X_2,\ldots,X_N)$, and calculating the result $y = f(x_1,x_2,\ldots,x_N)$.

7.8.1.6 Determine the Combined Standard Uncertainty, $u_c(y)$, of the Estimate, y

The CSU of y is obtained using the following formula:

$$u_c^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(x_i, x_j) \quad (7\text{-}32)$$

Here, $u^2(x_i)$ denotes the estimated variance of x_i, or the square of its standard uncertainty; $u(x_i,x_j)$ denotes the estimated covariance of x_i and x_j; $\partial f/\partial x_i$ (or $\partial y/\partial x_i$) denotes the partial derivative of f with respect to x_i evaluated at the measured values $x_1,x_2,\ldots,x_N$; and $u_c^2(y)$ denotes the combined variance of y, whose positive square root, $u_c(y)$, is the CSU of y. The partial derivatives, $\partial f/\partial x_i$, are called sensitivity coefficients, usually denoted c_i. The sensitivity coefficient measures how

much f changes when x_i changes. Equation 7-32 is called the "law of propagation of uncertainty" in GUM (ISO 1995).

If the input estimates $x_1,x_2,\ldots,x_N$ are uncorrelated, the uncertainty propagation formula reduces to

$$u_c^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) \qquad (7\text{-}33)$$

Suppose the values $x_1,x_2,\ldots,x_N$ are composed of two groups $w_1,w_2,\ldots,w_n$ and $z_1,z_2,\ldots,z_m$ with $N=n+m$. If all the variables, w and z, are uncorrelated and nonzero, the CSU of $y = \frac{w_1 w_2 \ldots w_n}{z_1 z_2 \ldots z_m}$ may be calculated from the formula:

$$u_c^2(y) = y^2 \left(\frac{u^2(w_1)}{w_1^2} + \frac{u^2(w_2)}{w_2^2} + \ldots + \frac{u^2(w_n)}{w_n^2} + \frac{u^2(z_1)}{z_1^2} + \frac{u^2(z_2)}{z_2^2} + \ldots + \frac{u^2(z_m)}{z_m^2} \right) \qquad (7\text{-}34)$$

The symbols $z_1,z_2,\ldots,z_m$ have been introduced simply to differentiate those values appearing in the denominator of the model equation from the $w_1,w_2,\ldots,w_n$ appearing in the numerator.

If $y = \frac{f(w_1, w_2, \ldots, w_n)}{z_1 z_2 \ldots z_m}$, where f is some specified function of $w_1,w_2,\ldots,w_n$, all the z_i are nonzero,

and all the input estimates are uncorrelated. Then:

$$u_c^2(y) = \frac{u_c^2(f(w_1, w_2, \ldots, w_n))}{z_1 z_2 \ldots z_m} + y^2 \left(\frac{u^2(z_1)}{z_1^2} + \frac{u^2(z_2)}{z_2^2} + \ldots + \frac{u^2(z_m)}{z_m^2} \right) \qquad (7\text{-}35)$$

An alternative to uncertainty propagation is the use of computerized Monte Carlo methods to propagate not the uncertainties of input estimates but their distributions. Given assumed distributions for the input estimates, the method provides an approximate distribution for the output estimate, from which the CSU or an uncertainty interval may be derived.

7.8.1.7 Optionally Multiply $u_c(y)$ by a Coverage Factor k to Obtain the Expanded Uncertainty U

The interval $[y - U, y + U]$, constructed using the expanded uncertainty $U = k \cdot u_c(y)$, can be expected to contain the value of the measurand with a specified probability, p. The specified probability, p, is called the "level of confidence" or the "coverage probability" and generally is only an approximation of the true probability of coverage. When the distribution of the measured result is approximately normal, the coverage factor often is chosen to be $k = 2$ for a coverage probability of approximately 95%. An expanded uncertainty calculated with $k = 2$ or 3 is sometimes informally called a "two-sigma" or "three-sigma" uncertainty, respectively. The GUM recommends the use of coverage factors in the range of 2 to 3 when the CSU represents a good estimate of the true standard deviation. Attachment 19D of MARLAP describes a more general procedure for calculating the coverage factor that gives a desired coverage probability p when there is substantial uncertainty in the value of $u_c(y)$.

7.8.1.8 Report the Result as $y \pm U$ with the Unit of Measurement

At a minimum, state the coverage factor used to compute U and the estimated coverage probability. Alternatively, report the result, y, and its CSU, $u_c(y)$, with the unit of measurement.

The number of significant figures that should be reported for the result of a measurement depends on the uncertainty of the result. A common convention, recommended by MARLAP, is to round the uncertainty (standard uncertainty or expanded uncertainty) to two significant figures and to report both the measured value and the uncertainty to the same number of decimal places. Only final results should be rounded in this manner. Intermediate results in a series of calculation steps should be carried through all steps with additional figures to prevent unnecessary round-off errors. Additional figures are also recommended when the data are stored electronically. Rounding should be performed only when the result is reported. Many of the values in the examples given in MARSAME carry more significant digits so that the calculations can be reasonably reproduced by the reader. All results, whether positive, negative, or zero, should be reported as obtained, together with their uncertainties.

A measured value y of a quantity Y that is known to be positive may be so far below zero that it indicates a possible blunder, procedural failure, or other quality control problem. Usually, if $y + 3u_c(y) < 0$, the result may be invalid. For example, if $y = -10$ and $u_c(y) = 1$, this would imply that Y is negative with high probability, which is known to be impossible. However, if $y = -1$ and $u_c(y) = 1$, the expanded uncertainty covers positive values with reasonable probability. The accuracy of the uncertainty estimate $u_c(y)$ must be considered in evaluating such results, especially in cases where only few counts are observed during the measurement and counting uncertainty is the dominant component of $u_c(y)$. (See MARLAP Chapter 18 and Attachment 19D).

7.8.2 Examples of Some Parameters that Contribute to Uncertainty

The sources of uncertainty described in the following sections, drawn from MARLAP Section 19.5, should be considered.

7.8.2.1 Instrument Background

Single-channel background measurements are usually assumed to follow the Poisson model, in which the uncertainty in the number of counts obtained, N, is given by $\sqrt{N}$. There may be effects that increase the variance beyond what the model predicts. For example, cosmic radiation and other natural sources of instrument background may vary between measurements, the instrument may become contaminated, or the instrument may simply be unstable. Generally, the variance of the observed background is somewhat greater than the Poisson counting variance, although for certain types of instruments, the Poisson model may overestimate the background variance (Currie et al., 1998). If the background does not closely follow the Poisson model, its variance should be estimated by repeated measurements.

The “instrument background,” or “instrument blank,” is usually measured under the same conditions that will be encountered in the field. Ambient background sources should be

minimized, and kept constant during the measurements of M&E. Periodic checks should be made to ensure that the instrument has not picked up additional radioactivity from the M&E during the measurements. If the background drifts or varies non-randomly over time (i.e., is non-stationary), it is important to minimize the consequences of the drift by performing frequent background measurements.

If repeated measurements demonstrate that the background level is stable, then the average, $\bar{x}$, the results of *n* similar measurements performed over a period of time may give the best estimate of the background. In this case, if all measurements have the same duration, the experimental standard deviation of the mean, $s(\bar{x})$, is also a good estimate of the measurement uncertainty.

Given the Poisson assumption, the best estimate of the uncertainty is still the Poisson estimate, which equals the square root of the summed counts, divided by the number of measurements, $\sqrt{n\bar{x}}/n = \sqrt{\bar{x}/n}$, but the experimental standard deviation may be used when the Poisson assumption is invalid. It is always wise to compare the value of $s(\bar{x})$ to the value of the Poisson uncertainty when possible to identify any discrepancies.

7.8.2.2 Counting Efficiency

The counting efficiency for a measurement of radioactivity (usually defined as the detection probability for a particle or photon of interest emitted by the source) may depend on many factors, including source geometry, placement, composition, density, activity, radiation type and energy and other instrument-specific factors. The estimated efficiency is sometimes calculated explicitly as a function of such variables (in gamma-ray spectroscopy, for example). In other cases a single measured value is used (e.g., alpha-particle spectrometry). If an efficiency function is used, the uncertainties of the input estimates, including those for both calibration parameters and sample-specific quantities, must be propagated to obtain the CSU of the estimated efficiency. Calibration parameters tend to be correlated; so, estimated covariances must also be included. If a single value is used instead of a function, the standard uncertainty of the value is determined when the value is measured. An example of the calculation of the uncertainty in counting efficiency is given in Example 16.

Example 16: A radiation counter is calibrated, taking steps to ensure that the geometry of the source position, orientation of the source, pressure, temperature, relative humidity, and other factors that could contribute to uncertainty are controlled, as described below:

The standard source is counted 15 times on the instrument for 300 s.
The radionuclide is long-lived; so, no decay corrections are needed. The uncertainties of the count times are assumed to be negligible.

Within the range of linearity of the instrument, the mathematical model for the calibration is:

$$\varepsilon = \frac{1}{n}\sum_{i=1}^{n}\frac{(N_{S,i}/t_S)-(N_B/t_B)}{a_s} \qquad (7\text{-}36)$$

Where:

ε = counting efficiency
n = number times the source is counted (15)
$N_{S,i}$ = gross count observed during the i^{th} measurement of the source

t_S = source count time (300 s)
N_B = observed background count (87)
t_B = background count time (6,000 s)
a_S = activity of the standard source (150.0 Bq). (The standard uncertainty of the source, 2.0 Bq, was given in the certificate for the source.)

The CSU of ε can be evaluated using Equation 7-36. For the purpose of uncertainty evaluation, it is convenient to rewrite the model as:

$$\varepsilon = \frac{\overline{R}}{a_s}$$

Where:

$$\overline{R} = \frac{1}{n}\sum_{i=1}^{n} R_i \quad \text{and} \quad R_i = (N_{S,i}/t_S) - (N_B/t_B), \quad i = 1,2,\ldots,n$$

The values R_i and their average, $\overline{R}$, are estimates of the count rate produced by the standard, while $\overline{R}/a_S$ is an estimate of the count rate produced by 1 Bq of activity. The standard uncertainty of $\overline{R}$ can be evaluated experimentally from the 15 repeated measurements: $u^2(\overline{R}) = s^2(\overline{R}) = \frac{1}{n(n-1)}\sum_{i=1}^{n}(R_i - \overline{R})^2$. Since only one background measurement was made, the input estimates R_i are correlated with each other. The uncertainty of N_B, $u(N_B) = \sqrt{87}$, using a Type B evaluation based on an assumption of a Poisson distribution for the number of background counts.

The covariance between R_i and R_j, for $i \neq j$, may be estimated as

$$u(R_i, R_j) = \frac{\partial R_i}{\partial N_B}\frac{\partial R_j}{\partial N_B}u^2(N_B) = \frac{-1}{t_B}\frac{-1}{t_B}u^2(N_B) = \frac{u^2(N_B)}{t_B^2} = \frac{\sqrt{87}^2}{6000^2} \cong 2\times 10^{-6}$$

However, the correlation is negligible here because the uncertainty of the background count, N_B, is much smaller than the uncertainty of each source count, $N_{S,i}$. So, the correlation of the input estimates R_i will be approximated as zero (i.e., treated as if they were uncorrelated), and the correlation terms dropped from Equation 7-32. This means the evaluation used to calculate the CSU of ε can proceed using equation 7-33.

$u_c^2(y) = \sum_{i=1}^{N}\left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i)$, so since $\varepsilon = \frac{\overline{R}}{a_s}$,

$$u_c^2(\varepsilon) = \left(\frac{\partial(\frac{\overline{R}}{a_s})}{\partial \overline{R}}\right)^2 u^2(\overline{R}) + \left(\frac{\partial(\frac{\overline{R}}{a_s})}{\partial a_s}\right)^2 u^2(a_s) = \left(\frac{1}{a_s}\right)^2 u^2(\overline{R}) + \left(\frac{-\overline{R}}{a_s^2}\right)^2 u^2(a_s)$$

$$= \left(\frac{u^2(\overline{R})}{a_s^2}\right) + \varepsilon^2\left(\frac{u^2(a_s)}{a_s^2}\right). \text{ Therefore, } u_c(\varepsilon) = \sqrt{\frac{u^2(\overline{R})}{a_S^2} + \varepsilon^2\frac{u^2(a_S)}{a_S^2}}$$

Assume the following data were obtained for the 15 separate counts of the calibration source.

Count Number, *i*	Gross count, $N_{s,i}$	R_i (s^{-1})
1	18,375	61.236
2	18,644	61.236
3	18,954	61.236
4	19,249	64.149
5	19,011	63.356
6	18,936	63.106
7	18,537	61.776
8	18,733	62.429
9	18,812	62.692
10	18,546	61.806
11	18,810	62.686
12	19,273	64.229
13	18,893	62.962
14	18,803	62.662
15	18,280	60.919
	Average, $\overline{R}$ (s^{-1})	62.6202
	Experimental standard deviation, $s(Ri)$ (s^{-1})	0.9483
	Experimental standard deviation of the mean, $s(\overline{R})$ (s^{-1})	0.2449

Then the estimated counting efficiency is:

$$\varepsilon = \frac{\overline{R}}{a_s} = \frac{62.6202\ \mathrm{s}^{-1}}{150.0\ \mathrm{Bq}} = 0.4176$$

And the CSU of ε is given by

$$u_c(\varepsilon) = \sqrt{\frac{(0.2449\ \mathrm{s}^{-1})^2}{(150.0\ \mathrm{Bq})^2} + 0.4176^2 \times \frac{(2.0\ \mathrm{Bq})^2}{(150.0\ \mathrm{Bq})^2}} = 0.005802$$

Which may be rounded to 0.0058.

The true counting efficiency may vary because of variations in geometry, position and other influence quantities not explicitly included in the model. These sources of uncertainty may not be controlled as they were in the above example. If this is the case, the standard uncertainty of ε should include not only the standard uncertainty of the estimated mean, as calculated in the example, but also another component of uncertainty due to variations of the true efficiency during subsequent measurements. The additional component may be written as $\varepsilon\phi$, where ϕ is the coefficient of variation (i.e., the standard deviation divided by the mean) of the true efficiency. Then the total uncertainty of ε is obtained by squaring the original uncertainty estimate, adding $\varepsilon^2\phi^2$, and taking the square root of the sum.

$$u_c(\varepsilon) = \sqrt{\frac{u^2(\overline{R})}{a_S^2} + \varepsilon^2\left(\frac{u^2(a_S)}{a_S^2} + \phi^2\right)} \quad (7\text{-}37)$$

In the example above, the experimental variance of the count rates, R_i, may be used to estimate ϕ. Section 18B.2 of Attachment 18B of MARLAP describes an approach for estimating such "excess" variance in a series of measurements.

Variations in counting efficiency due to source placement should be reduced as much as possible through the use of positioning devices that ensure a source with a given geometry is always placed in the same location relative to the detector. If such devices are not used, variations in source position may significantly increase the measurement uncertainty.

Calibrating an instrument under conditions different from the conditions under which M&E sources are counted may lead to large uncertainties in the activity measurements. Source geometry in particular tends to be an important factor for many types of radiation counters. If correction factors are used, their uncertainties should be evaluated and propagated, as mentioned in Section 7.8.1.1.

7.8.2.3 Digital Displays and Rounding

If a measuring device has a digital display with readability[10] δ, the standard uncertainty of a measured value is at least $\delta / 2\sqrt{3}$, which is the variance of a random variable uniformly distributed over the interval $(x - \delta/2, x + \delta/2)$. Note that this is the same result as given by equation 7-24 with $a = \delta/2$. This uncertainty component exists even if the instrument is completely stable.

A similar Type B method may be used to evaluate the standard uncertainty due to computer round-off error. When a value x is rounded to the nearest multiple of 10^n (where n is an integer), the component of uncertainty generated by round-off error is $10^n/(2\sqrt{3})$. This component of uncertainty should be kept small in comparison to the total uncertainty of x by performing rounding properly and printing with an adequate number of figures. In a long calculation involving mixed operations, carry as many digits as possible through the entire set of calculations and then round the final result appropriately as described in MARLAP Section 19.3.7 (MARLAP 2004).

Example 17: The readability of a digital survey dose rate meter is 1 nGy/h. Therefore, the minimum standard uncertainty of a measured absorbed dose rate is $1/2\sqrt{3}$ = 0.29 nGy/h.

[10] Readability is the smallest difference that can still be read on a display. For instruments with an analog indicating device, the readability is equal to the smallest fraction of a scale interval that can still be estimated with reasonable reliability or which can be determined by an auxiliary device. For instruments with a numeric indicator (digital display), the readability is equal to one digital step.

Example 18: Suppose the results for R_i in Example 16 had been rounded to the nearest whole number before the analysis. Then the average would be computed as 62.6 instead of 62.6202 and the standard deviation would be computed as 0.9103 instead of 0.9483. This demonstrates the effect that rounding intermediate results can have on subsequent calculations. If this rounding to the nearest positive integer had already occurred prior to receiving the data, and the original data were no longer available, a correction for it could be made when estimating the CSU of R_i. The component of uncertainty generated by round-off error is $1/(2\sqrt{3})$:

$$u(R_i) = \sqrt{0.9103^2 + \left(\frac{1}{2\sqrt{3}}\right)^2} = 0.9549.$$

7.8.3 Example Uncertainty Calculation

To illustrate how the uncertainty calculations are performed in practice, the following example is given based on that of Lewis et al. (2005). The calculation will be that of the CSU in the calibration of a surface contamination monitor.

7.8.3.1 Model Equation and Sensitivity Coefficients

Surface contamination monitors are calibrated in terms of their response to known rates of radioactive emissions. In practice this is achieved by using large-area, planar sources that have a defined area and whose emission rates have been determined in a traceable manner. The calibration is usually determined in terms of response per emission rate per unit area. In this example, the source is positioned with its active face parallel to and at a distance of 3 mm from the face of the detector. The monitor detector area (50 cm^2) is smaller than the area of the calibration source, which is a 10 cm × 10 cm layer of ^{14}C on a thick aluminum substrate. The monitor has an analog display and has a means to set the detector voltage.

The efficiency, ε, is defined by:

$$\varepsilon = \frac{(M - B) \times f_V \times f_d \times f_u \times f_{bs}}{\left(E \diagup A\right)} \tag{7-38}$$

Where:

M = observed monitor reading, s^{-1}
B = background reading, s^{-1}
E = emission rate of the calibration source, s^{-1}
A = area of the active portion of the calibration source, cm^2
f_V = plateau voltage factor
f_d = source-detector separation factor
f_u = source uniformity factor
f_{bs} = backscatter factor

The sensitivity coefficients of Equation 7-38 are given by:

$$\frac{\partial \varepsilon}{\partial M} = (A/E) \times f_V \times f_d \times f_u \times f_{bs} = \frac{\varepsilon}{(M - B)} \tag{7-39}$$

$$\frac{\partial \varepsilon}{\partial B} = -(A/E) \times f_V \times f_d \times f_u \times f_{bs} = \frac{-\varepsilon}{(M-B)} \quad (7\text{-}40)$$

$$\frac{\partial \varepsilon}{\partial E} = -(M-B)(A/E^2) \times f_V \times f_d \times f_u \times f_{bs} = \frac{-\varepsilon}{E} \quad (7\text{-}41)$$

$$\frac{\partial \varepsilon}{\partial A} = (M-B)(1/E) \times f_V \times f_d \times f_u \times f_{bs} = \frac{\varepsilon}{A} \quad (7\text{-}42)$$

$$\frac{\partial \varepsilon}{\partial f_V} = (M-B)(A/E) \times f_d \times f_u \times f_{bs} = \frac{\varepsilon}{f_V} \quad (7\text{-}43)$$

$$\frac{\partial \varepsilon}{\partial f_d} = (M-B)(A/E) \times f_V \times f_u \times f_{bs} = \frac{\varepsilon}{f_d} \quad (7\text{-}44)$$

$$\frac{\partial \varepsilon}{\partial f_u} = (M-B)(A/E) \times f_V \times f_d \times f_{bs} = \frac{\varepsilon}{f_u} \quad (7\text{-}45)$$

$$\frac{\partial \varepsilon}{\partial f_{bs}} = (M-B)(A/E) \times f_V \times f_d \times f_u = \frac{\varepsilon}{f_{bs}} \quad (7\text{-}46)$$

Under normal conditions, the factors f_V, f_d, f_u and f_{bs} are each assumed to have a value of one. If the uncertainties are to be calculated in relative terms, the uncertainty equation becomes (see Equation 7-34):

$$\left(\frac{\sigma_C}{\varepsilon}\right)^2 = \left(\frac{M}{M-B}\right)^2 \left(\frac{\sigma_M}{M}\right)^2 + \left(\frac{B}{M-B}\right)^2 \left(\frac{\sigma_B}{B}\right)^2 + \left(\frac{\sigma_E}{E}\right)^2 + \left(\frac{\sigma_A}{A}\right)^2 + \left(\frac{\sigma_{f_V}}{f_V}\right)^2 + \left(\frac{\sigma_{f_d}}{f_d}\right)^2 + \left(\frac{\sigma_{f_u}}{f_u}\right)^2 + \left(\frac{\sigma_{f_{bs}}}{f_{bs}}\right)^2 \quad (7\text{-}47)$$

If the relative uncertainties are all expressed as percentages, $\left(\frac{\sigma_{x_i}}{x_i}\right)$, where x_i is an input quantity, then the CSU will be a percentage. The relative sensitivity coefficients, c_i, are the terms multiplying each relative uncertainty term $\left(\frac{\sigma_{x_i}}{x_i}\right)$ in Equation 7-47. This approach produces relative sensitivity coefficients of unity for the last 6 terms.

7.8.3.2 Uncertainty Components

Monitor Reading of Source, *M* (Type A)

Several techniques can be used to determine the mean observed monitor reading, *M*, and its uncertainty. Assume a snap-shot technique is used whereby six successive, but randomly timed, readings are recorded, giving 350, 400, 400, 325, 350, 350 s^{-1}. The mean and standard deviation

of the mean becomes 362.5 ± 12.5 s^{-1}. This equates to a percentage uncertainty in *M* of 3.45% and the relative sensitivity coefficient from Equation 7-47, $\frac{M}{(M-B)}$, is 362.5/(362.5 – 32.5), which is equal to 1.10. The distribution is assumed to be normal.

Monitor Reading of Background, *B* (Type B)

In this case, an eye-averaging technique was used whereby the highest and lowest count rates were recorded over a given period of time. These count rates were 40 and 25 s^{-1} respectively, giving a mean value of 32.5 s^{-1}. This value is assumed to have a rectangular distribution with a half-width of 7.5 s^{-1}, and an uncertainty of $7.5/\sqrt{3} = 4.330$, equating to a percentage uncertainty of $4.330/32.5 = 0.1332$ or 13%. The relative sensitivity coefficient from Equation 7-47, $\frac{B}{(M-B)}$, is 32.5/(362.5 - 32.5), which gives a value of 0.098.

Emission Rate of Calibration Source, *E* (Type B)

The emission rate of the source and its uncertainty were provided on the calibration certificate by the laboratory that calibrated the source using a windowless proportional counter. The statement on the certificate was:

"The measured value of the emission rate is E = 2,732±13 s^{-1}"

The reported uncertainty is based on a standard uncertainty multiplied by a coverage factor of *k* = 2, which provides a level of confidence of approximately 95%. The standard uncertainty on *E* is therefore 13/2 = 6.5 s^{-1} or 0.24%. Unless the certificate provides information to the contrary, it is assumed that the uncertainty has a normal distribution.

Source Area, *A* (Type B)

In the absence of an uncertainty statement by the manufacturer, the only information available is the product drawing that shows the active area dimensions to be 10 cm × 10 cm. On the assumption that the outer bounds of the length, *L*, and the width, *W*, are 9.9 and 10.1 cm, the uncertainty of the linear dimensions may be taken to be a rectangular distribution with a half-width of 0.1 cm.

L = 10 and *u(L)* = $0.1/\sqrt{3} = 0.0577$. *W* = 10 and *u(W)* = $0.1/\sqrt{3} = 0.0577$. Because *A* = *LW*, we get $u^2(A) = u^2(LW) = L^2u^2(W) + W^2u^2(L) = 2(10)^2(0.0577)^2 = 0.665858$, therefore $u(A) = 0.816\,\text{cm}^2$ or 0.816%.

Plateau Voltage Factor, f_V (Type B)

This applies only to those instruments where voltage adjustments are possible. If the setting is not checked and/or adjusted between calibrations, then this has no effect. Changing the plateau voltage without performing a recalibration is not recommended. If, however, the user is allowed to do this, the setting may not be returned to exactly that used during the calibration. In this particular example, the slope of the response curve in this region is taken to be 10% / 50 v. It is

assumed that an operator is more likely to set the voltage nearer to the optimum than the extremes and that ± 50 v represents the range at the 100% confidence level. Accordingly, a triangular distribution is assumed with a half-width of 50 v, equating to an uncertainty for the voltage of $50/\sqrt{6} = 20.4124$ and an uncertainty for the voltage factor of 20.4124(10%)/ 50 = 4.0825%.

Source-Detector Separation Factor, f_d (Type B)

This effect arises from the uncertainty in mounting the calibration source exactly 3 mm from the detector face. Experimental evidence has shown that, for the particular ^{14}C source at 3 mm source-detector separation, the change in response was 2.6% per mm. It is assumed that the deviation from the nominal 3-mm separation is no greater than 1 mm but that all values are equally probable between 2 and 4 mm, a rectangular distribution. The uncertainty in the separation is thus $1/\sqrt{3} = 0.5774$. The uncertainty of the separation factor is thus 0.5774 mm × 2.6% / mm, equal to 1.5011%.

Non-Uniformity of Calibration Source, f_u (Type B)

Large area sources may have a non-uniform activity distribution across their surfaces. For the ^{14}C source, the uniformity is assumed to be better than ± 10%. This is based on comparing 10 cm^2 sections of the source. For a typical monitor with a detector area of 50 cm^2 and a calibration source area of 100 cm^2, a worst-case condition could be that the area under the detector has an activity per unit area that is 10% greater than the mean value for the whole source. (The outer area correspondingly will be 10% less than mean value.) Assuming a rectangular distribution, this represents an uncertainty of $10/\sqrt{3} = 5.774\%$ for the source non-uniformity factor.

Backscatter Factor, f_{bs} (Type B)

Variations in backscatter effects arise from factors such as the nature of the surface on which the calibration source is resting and the proximity to scattering surfaces such as walls. This effect can be quite marked for photon emitters, but for ^{14}C on aluminum substrates the effect is negligible.

7.8.3.3 Uncertainty Budget

An important part of the uncertainty analysis is to determine which factors are contributing the most to the overall uncertainty.

To do this, each component of uncertainty $u_i(y)=c_i\, u_i(x_i)$ is squared to give its component of variance $(u_i(y))^2$. These are totaled to obtain the total variance, 69.07. Finally, the ratio of each component of variance to the total is computed. The relative sensitivity coefficients, c_i, are the terms multiplying each relative uncertainty term $\left(\frac{\sigma_{x_i}}{x_i}\right)$ in Equation 7-47.

The last column of the uncertainty budget (Table 7.7) shows that the major source of uncertainty is due to source non-uniformity (48%) followed by the voltage factor (24%) and the reading of the source (21%). Thus, to decrease the overall uncertainty, attention should be paid to those factors first.

Table 7.7 Uncertainty Budget for the Efficiency Example

Source of Uncertainty	Type	Probability Distribution	*Relative Sensitivity Coeffient,c_i*	$u_i(x_i)$ (%)	$u_i(y)= c_i\, u_i(x_i)$ (%)	$(u_i(y))^2$	$(u_i(y))^2$/Total
Standard deviation of mean of *M*	A	Normal	1.10	3.45	3.80	14.44	0.21
Standard deviation of mean of *B*	B	Rectangular	0.098	13.32	1.31	1.72	0.02
Standard uncertainty of calibration source emission rate, *E*	B	Normal	1.0	0.24	0.24	0.06	0.00
Half -width of source length, L and width W on the area A	B	Product of 2 independent rectangular	1.0	0.816	0.816	0.666	0.01
Half -width of voltage factor, f_V	B	Triangular	1.0	4.08	4.08	16.65	0.24
Half -width of source-detector separation factor, f_d	B	Rectangular	1.0	1.50	1.50	2.25	0.03
Half-width of calibration source non- uniformity factor, f_u	B	Rectangular	1.0	5.77	5.77	33.29	0.48
Uncertainty of backscatter factor, f_{bs}	B	n.a.	1.0	0.0	0.0	0.00	0.00
Combined standard uncertainty		Normal	—	—	8.31 = $\sqrt{69.07}$	Total= 69.07	0.99
Expanded uncertainty (k=2)		Normal	—	—	2·8.31= 16.6	—	—

7.8.3.4 Reported Result

Using the formula above, the calibration factor in terms of emission rate becomes:

$$\varepsilon = \frac{(M-B)\times f_V \times f_d \times f_u \times f_{bs}}{\left(E/A\right)} = \frac{(362.5-32.5)\times 1\times 1\times 1\times 1}{\left(2732/100\right)} = 12.1 \text{ (counts} \times \text{s}^{-1})/(\text{s}^{-1} \times \text{cm}^{-2}) \qquad (7\text{-}48)$$

The CSU is (12.1)×(0.0831) = 1.0056. The reported expanded uncertainty will be 2.0, based on a standard uncertainty of 1.0 multiplied by a coverage factor of $k = 2$, which provides a level of confidence of approximately 95%.

7.9 Calculate the Minimum Detectable Concentration

This section is intended to expand on the material in Section 7.5. It contains more statistical detail and more complex examples. This advanced material may be deferred on a first reading of MARSAME.

7.9.1 Critical Value

In the terminology of ISO 11843-1 (1997), the measured concentration is the state variable, denoted by Y, which represents the state of the material being analyzed. The state variable usually cannot be observed directly, but it is related to an observable response variable, denoted by X, through a calibration function F, the mathematical relationship being written as $X = F(Y)$. The response variable X is most often an instrument signal, such as the number of counts observed. The inverse, $Y = F^{-1}(X)$ of the calibration function is sometimes called the evaluation function. The evaluation function, which gives the value of the net concentration in terms of the response variable, is closely related to the mathematical model $Y = f(X_1, X_2, \ldots, X_N)$ described in Section 7.8.1.1.

Either the null or the alternative hypothesis is chosen on the basis of the observed value of the response variable, X. The value of X must exceed a certain threshold value to justify rejection of the null hypothesis and acceptance of the alternative hypothesis. This threshold is called the critical value of the response variable and is denoted by x_C.

The calculation of x_C requires the choice of a significance level for the test. The significance level is a specified upper bound for the probability, α, of a Type I error. The significance level is usually chosen to be 0.05. This means that when there is no radiation or radioactivity present (above background), there should be at most a 5% probability of incorrectly deciding that it is present.

The critical value of the concentration, y_C, is defined as the value obtained by applying the evaluation function, F^{-1}, to the critical value of the response variable, x_C. Thus, $y_C = F^{-1}(x_C)$. When x is the gross instrument signal, this formula typically involves subtraction of the background signal and division by the counting efficiency, and possibly other factors.
A detection decision can be made by comparing the observed gross instrument signal to its critical value, x_C, as indicated above. However, it has become standard practice to make the decision by comparing the net instrument signal to its critical value, S_C. The net signal is calculated from the gross signal by subtracting the estimated blank value and any interference. The critical value of the net instrument signal, S_C, is calculated from the critical gross signal, x_C, by subtracting the same correction terms; so, in principle, either approach should lead to the same detection decision.

Because the term "critical value" alone is ambiguous, one should specify the variable to which the term refers. For example, one may discuss the critical (value of the) radionuclide concentration, the critical (value of the) net signal, or the critical (value of the) gross signal. In this document, the signal is usually a count, and the critical value generally refers to the net count.

The response variable is typically an instrument signal, whose mean value generally is positive even when there is radioactivity present (i.e., above background). The gross signal must be corrected by subtracting an estimate of the signal produced under those conditions. See Section 7.8.2.1 (Instrument Background).

7.9.2 Minimum Detectable Concentration

The minimum detectable concentration (MDC) is the minimum concentration of radiation or radioactivity that must be present in a sample to give a specified power, $1 - \beta$. It may also be defined as:

- The minimum radiation or radioactivity concentration that must be present to give a specified probability, $1 - \beta$, of detecting the radiation or radioactivity; or
- The minimum radiation or radioactivity concentration that must be present to give a specified probability, $1 - \beta$, of measuring a response greater than the critical value, leading one to conclude correctly that there is radiation or radioactivity present.

The *power* of any hypothesis test is defined as the probability that the test will reject the null hypothesis when it is false, i.e., the correct decision. Therefore, if the probability of a Type II error is denoted by β, the power is $1 - \beta$. In the context of radiation or radioactivity detection, the power of the test is the probability of correctly detecting the radiation or radioactivity (concluding that the radiation or radioactivity is present), which happens whenever the response variable exceeds its critical value. The power depends on the concentration of the radiation or radioactivity and other conditions of measurement; so, one often speaks of the "power function" or "power curve." Note that the power of a test for radiation or radioactivity detection generally is an increasing function of the radiation or radioactivity concentration (i.e., the greater the radiation or radioactivity concentration, the higher the probability of detecting it).

In the context of MDC calculations, the value of β that appears in the definition, like α, is usually chosen to be 0.05 or is assumed to be 0.05 by default if no value is specified. The minimum detectable concentration is denoted in mathematical expressions by y_D. The MDC is usually obtained from the minimum detectable value of the net instrument signal, S_D. S_D, is defined as the mean value of the net signal that gives a specified probability, $1 - \beta$, of yielding an observed signal greater than its critical value S_C. The relationship between the critical value of the net instrument signal, S_C, and the minimum detectable net signal, S_D, is shown in Figure 7.6 in Section 7.5.2.

The term MDC must be carefully and precisely defined to prevent confusion. The MDC is by definition an estimate of the true concentration of the radiation or radioactivity required to give a specified high probability that the measured response will be greater than the critical value.

The common practice of comparing a measured concentration to the MDC, instead of to the S_C, to make a detection decision is incorrect. If this procedure were used, then there would be only a a 50% chance of deciding that radioactivity was present when the concentration was actually at the MDC. This is in direct contradiction to the definition of MDC. See MARLAP Appendix B, Attachment B1 for a further discussion of this issue.

Since the MDC is calculated from measured values of input quantities such as the counting efficiency and background level, the MDC estimate has a CSU, which in principle can be obtained by uncertainty propagation. To avoid confusion, it may be useful to remember that a detection decision is usually made by comparing the instrument response to the critical value, and that the critical value generally does not even have the units of radiation or radioactivity concentration.

7.9.3 Calculation of the Critical Value

If the net signal is a count, then in many circumstances the uncertainty in the count can be estimated by a Type B evaluation using the fact that for a Poisson distribution with mean N_B, the variance is also N_B. Thus, the uncertainty in the background count is estimated as $\sqrt{N_B}$ and the critical value is often an expression involving $\sqrt{N_B}$.

The most commonly used approach for calculating the critical value of the net instrument signal, S_C, is given by the following equation.[11]

$$S_C = z_{1-\alpha}\sqrt{N_B \frac{t_S}{t_B}\left(1+\frac{t_S}{t_B}\right)} \tag{7-49}$$

Where:

N_B = background count
t_S = count time for the sample
t_B = count time for the background
$z_{1-\alpha}$ = $(1-\alpha)$-quantile of the standard normal distribution

Example 19: A 6,000-second background measurement is performed on a proportional counter and 108 beta counts are observed. A sample is to be counted for 3,000 s. Estimate the critical value of the net count when $\alpha = 0.05$.

$$S_C = z_{1-\alpha}\sqrt{N_B \frac{t_S}{t_B}\left(1+\frac{t_S}{t_B}\right)}$$

[11] This expression for the critical net count depends for its validity on the assumption of Poisson counting statistics. If the variance of the blank signal is affected by interferences, or background instability, then Equation 20.7 of MARLAP may be more appropriate. Interference is the presence of other radiation or radioactivity or electronic signals that hinder the ability to analyze for the radiation or radioactivity of interest.

$$S_C = 1.645\sqrt{108 \times \left(\frac{3{,}000\text{ s}}{6{,}000\text{ s}}\right)\left(1+\frac{3{,}000\text{ s}}{6{,}000\text{ s}}\right)} = 14.8 \text{ net counts}$$

If α = 0.05 and $t_B = t_S$, Equation 7-49 leads to the well-known expression $2.33\sqrt{N_B}$ for the critical net count (Currie, 1968).

When the background count is high (e.g., 100 or more), Equation 7-49 works well, but at lower background levels it can produce a high rate of Type I errors. Because this is a Scenario B hypothesis test, this means that too often a decision will be made that there is radiation or radioactivity present when it actually is not.

When the mean background counts are low and $t_B \neq t_S$, another approximation formula for S_C appears to out-perform all of the other approximations reviewed in MARLAP, namely the Stapleton approximation:

$$S_C = d \times \left(\frac{t_S}{t_B} - 1\right) + \frac{z_{1-\alpha}^2}{4} \times \left(1 + \frac{t_S}{t_B}\right) + z_{1-\alpha}\sqrt{\left(N_B + d\right)\frac{t_S}{t_B}\left(1+\frac{t_S}{t_B}\right)} \quad (7\text{-}50)$$

When $\alpha = 0.05$, setting the parameter $d = 0.4$ yields the best results. When, in addition, $t_B = t_S$, the Stapleton approximation gives the equation

$$S_C = 1.35 + 2.33\sqrt{N_B + 0.4} \quad (7\text{-}51)$$

7.9.4 Calculation of the Minimum Detectable Value of the Net Instrument Signal

The traditional method for calculating the MDC involves three steps: first calculating critical value of the net instrument signal, then calculating the minimum detectable value of the net instrument signal and finally converting the result to a concentration using the mathematical measurement model.

The minimum detectable value of the net instrument signal, denoted by S_D, is defined as the mean value of the net signal that gives a specified probability, $1 - \beta$, of yielding an observed signal greater than its critical value, S_C.

Note: The MDC may be estimated by calculating the minimum detectable value of the net instrument signal, S_D, and converting the result to a concentration.

Counting data rarely, if ever, follow the Poisson model exactly, but the model can be used to calculate S_D if the variance of the background signal is approximately Poisson and a conservative value of the efficiency constant, ε, is used to convert S_D to y_D. The equation below shows how to calculate S_D using the Poisson model.

$$S_D = S_C + \frac{z_{1-\beta}^2}{2} + z_{1-\beta}\sqrt{\frac{z_{1-\beta}^2}{4} + S_C + R_B t_S\left(1+\frac{t_S}{t_B}\right)} \quad (7\text{-}52)$$

Where:

S_C = critical value

R_B = mean count rate of the blank, $R_B = \frac{N_B}{t_B}$

N_B = background count

t_S = count time for the test source

t_B = count time for the background

$z_{1-\beta}$ = $(1-\beta)$-quantile of the standard normal distribution

When Equation 7-49 is appropriate for the critical net count, and $\alpha = \beta$, this expression for S_D simplifies to $z_{1-\beta}^2 + 2S_C$. If in addition, $\alpha = \beta = 0.05$ and $t_B = t_S$ then

$$S_D = 2.71 + 2S_C = 2.71 + 2(2.33\sqrt{N_B}) = 2.71 + 4.66\sqrt{N_B} \quad (7\text{-}53)$$

Example 20: A 6,000-s background measurement on a proportional counter produces 108 beta counts and a source is to be counted for 3,000 s. Assume the background measurement gives the available estimate of the true mean background count rate, R_B and use the value 0.05 for Type I and Type II error probabilities. From Section 7.9.3, Example 19, the critical net count, S_C, equals 14.8, so $S_D = z_{1-\beta}^2 + 2S_C = 1.645^2 + 2\,(14.8) = 32.3$ net counts.

When the Stapleton approximation (Equation 7-51) is used for S_C, the minimum detectable net count S_D may be calculated using the Equation 7-53, but when the Poisson model is assumed, a better estimate is given by the equation:

$$S_D = \frac{(z_{1-\alpha} + z_{1-\beta})^2}{4}\left(1 + \frac{t_S}{t_B}\right) + (z_{1-\alpha} + z_{1-\beta})\sqrt{R_B t_S\left(1 + \frac{t_S}{t_B}\right)} \quad (7\text{-}54)$$

This equation is the same as that recommended by ISO 11929-1 (ISO 2000) in a slightly different form.

When $\alpha = \beta = 0.05$ and $t_B = t_S$, the preceding equation becomes:

$$S_D = 5.41 + 4.65\sqrt{R_B t_S} \quad (7\text{-}55)$$

Consult MARLAP Chapter 20 for a discussion of the calculation of S_D and y_D when both Poisson counting statistics and other sources of variance are considered.

7.9.5 Calculation of the Minimum Detectable Concentration

The MDC is often used to compare different measurement procedures against specified requirements. The calculation of the nominal MDC is complicated by the fact that some input quantities in the mathematical model, such as interferences, counting efficiency, and instrument background may vary significantly from measurement to measurement. Because of these variable quantities, determining the value of the radiation or radioactivity concentration that

corresponds to the minimum detectable value of the net instrument signal, S_D, may be difficult in practice. One common approach to this problem is to make conservative choices for the values of the variable quantities, which tend to increase the value of the MDC.

The mean net signal, S, is usually directly proportional to Y, the true radiation or radioactivity concentration present. Hence, there is a efficiency constant, ε, such that $S = \varepsilon Y$. The constant ε is typically the mean value of the product of factors such as the source count time, decay-correction factor, and counting efficiency. Therefore, the value of the minimum detectable concentration, y_D, is

$$y_D = \frac{S_D}{\varepsilon} \tag{7-56}$$

The preceding equation is only true if all sources of variability are accounted for when determining the distribution of the net signal, $\hat{S}$. Note that ensuring the MDC is not underestimated also requires that the value of ε not be overestimated.

Using any of the equations in Section 7.5.2 to calculate S_D is only appropriate if a conservative value of the efficiency constant, ε, is used when converting S_D to the MDC.

Example 21: Consider a scenario where t_B = 6,000 s, t_S = 3,000 s, and $R_B \approx 0.018$ s^{-1}. Let the measurement model be $Y = \frac{N_S - (N_B t_S / t_B)}{t_S \varepsilon}$

Where:

Y = activity of the radionuclide in the sample and

ε = counting efficiency (counts per second)/(Bq/cm^2)

Assume the source count time, t_S, has negligible variability, the counting efficiency has mean 0.42 and a 10% relative CSU, and from Example 20, S_D = 32.3 net counts.

The mean minimum detectable concentration is $y_D = \frac{S_D}{t_S \varepsilon} = \frac{32.3}{(3000)(0.42)} = 0.0256$ Bq/cm^2.

Adjusting for the 10% variability in the counting efficiency, the uncertainty is (0.10)×(0.42) = 0.042. Assuming that the efficiency is normally distributed, the lower 5th percentile for ε is (0.42)−(1.645)(0.042) = 0.35, where −1.645 is the 5th percentile of a standard normal distribution. Therefore, a conservative estimate of the efficiency constant is ε = 0.35 and a conservative estimate of the minimum detectable concentration is:

$y_D = \frac{S_D}{t_S \varepsilon} = \frac{32.3}{(3000)(0.35)} = 0.0308$ Bq/cm^2.

An alternative procedure could be to recognize that because of the uncertainties in the input estimates entered into the measurement model to convert from S_D to Y, that the MDC is actually a random variable. Then the methods for propagation of uncertainty given in Section 7.8 can be applied. Using the same assumptions as above, we would find that $y_D = 0.0256 \pm 0.0051$ with 95% confidence based on a coverage factor of 2. Therefore the 95% upper confidence level for y_D would be 0.0307 Bq.

More conservative (higher) estimates of the MDC may be obtained by following NRC recommendations (NRC 1984), in which formulas for the MDC include estimated bounds for relative systematic error in the background determination ($\blacktriangle_B$) and the sensitivity ($\blacktriangle_A$). The critical net count S_C is increased by $\blacktriangle_B N_B (t_S/t_B)$, and the minimum detectable net count S_D is increased by $2\blacktriangle_B N_B (t_S/t_B)$. Next, the MDC is calculated by dividing S_D by the efficiency and multiplying the result by $1+\blacktriangle_A$. The conservative approach presented in NRC 1984 treats random errors and systematic errors differently to ensure that the MDC for a measurement process is unlikely to be consistently underestimated, which is an important consideration if it is required by regulation or contract to achieve a specified MDC.

7.10 Calculate the Minimum Quantifiable Concentration

This section is intended to expand on the material in Section 7.6. It contains more statistical detail and more complex examples. This advanced material may be deferred on a first reading of MARSAME.

Calculation of the MQC requires that one be able to estimate the standard deviation for the result of a hypothetical measurement performed on a sample with a specified radionuclide concentration. The MQC is defined symbolically as the value y_Q that satisfies the relation:

$$y_Q = k_Q \sqrt{\sigma^2(y \mid Y = y_Q)} \tag{7-57}$$

Where the specified relative standard deviation of y_Q is $1/k_Q$ (usually chosen to be 10% so that $k_Q = 10$). $\sigma^2(y \mid Y = y_Q)$ is the variance of the estimator y given the true concentration Y equals y_Q. If the function $\sigma^2(y \mid Y = y_Q)$ has a simple form, it may be possible to solve the above equation for y_Q using only algebraic manipulation. Otherwise, fixed-point iteration, or other more general approaches, may be used, as discussed in MARLAP Section 20.4.3.

When Poisson counting statistics are assumed, and the mathematical model for the radionuclide concentration is

$$Y = S/\varepsilon \tag{7.58}$$

Where:

S	=	net count
t_S	=	count time for the source
S/t_S	=	net count rate
ε	=	efficiency of the measurement

Then Equation 7-57 may be solved for y_Q to obtain:

$$y_Q = \frac{k_Q^2}{2t_S\varepsilon(1-k_Q^2\phi_{\hat{\varepsilon}}^2)}\left(1+\sqrt{1+\frac{4(1-k_Q^2\phi_{\hat{\varepsilon}}^2)}{k_Q^2}\left(R_B t_S\left(1+\frac{t_S}{t_B}\right)+R_I t_S+\sigma^2(\hat{R}_I)t_S^2\right)}\right) \quad (7\text{-}59)$$

Where:

t_S = count time for the source, s
t_B = count time for the background, s
R_B = mean background count rate, s^{-1}
R_I = mean interference count rate, s^{-1}
$\sigma(\hat{R}_I)$ = standard deviation of the measured interference count rate, s^{-1}, and
$\phi_{\hat{\varepsilon}}^2$ = relative variance of the measured efficiency, $\hat{\varepsilon}$

If the efficiency ε may vary, then a conservative value, such as the 0.05-quantile $\varepsilon_{0.05}$, should be substituted for ε in the formula. Note that $\phi_{\hat{\varepsilon}}^2$ denotes only the relative variance of $\hat{\varepsilon}$ due to subsampling and measurement error; it does not include any variance of the efficiency ε itself (see discussion in Section 7.8).

Note that Equation 7-59 defines the MQC only if $1-k_Q^2\phi_{\hat{\varepsilon}}^2 > 0$. If $1-k_Q^2\phi_{\hat{\varepsilon}}^2 \le 0$, the MQC is infinite, because there is no concentration at which the relative standard deviation of *y* fails to exceed $1 / k_Q$. In particular, if the relative standard deviation of the measured efficiency $\hat{\varepsilon}$ exceeds $1 / k_Q$, then $1-k_Q^2\phi_{\hat{\varepsilon}}^2 < 0$ and the MQC is infinite.

If there are no interferences, Equation 7-59 simplifies to:

$$y_Q = \frac{k_Q^2}{2t_S\varepsilon(1-k_Q^2\phi_{\hat{\varepsilon}}^2)}\left(1+\sqrt{1+\frac{4(1-k_Q^2\phi_{\hat{\varepsilon}}^2)}{k_Q^2}\left(R_B t_S\left(1+\frac{t_S}{t_B}\right)\right)}\right) \quad (7\text{-}60)$$

Example 22: Consider the scenario of Example 21, where t_B = 6,000 s, t_S = 3,000 s, and $R_B \approx 0.018$ s^{-1}. Suppose the measurement model is $Y = \frac{N_S - (N_B t_S / t_B)}{t_S\varepsilon}$

Where:

Y = specific activity of the radionuclide in the sample
ε = counting efficiency (cps/Bq)/(Bq/cm^2)

Assume the source count time, t_S, has negligible variability, the counting efficiency has a mean of 0.42 and a 5% relative CSU, and S_D = 32.3 net counts.

S_D / t_S =32.3/3000 is the net count rate and the counting efficiency, ε, is 0.42.

The mean minimum detectable concentration is $y_D = \frac{S_D}{t_S\varepsilon} = \frac{32.3}{(3000)(0.42)} = 0.0256$ Bq/cm^2.

Also assume:

k_Q = 10

$\phi_{\hat{\varepsilon}} = 0.05$

$\phi_{\hat{\varepsilon}}^2 = 0.05^2$

$1 - k_Q^2 \phi_{\hat{\varepsilon}}^2 = 1 - 100 \times (0.05^2) = 0.75$

There are no interferences so that Equation 7-60 can be used.

Note that if the counting efficiency had a mean of 0.42 and a 10% relative standard uncertainty as in Example 11, then $1 - k_Q^2 \phi_{\hat{\varepsilon}}^2 = 1 - 100 \times (0.10^2) = 0$ and the MQC would be infinite. Therefore it was necessary to change the procedure for evaluating the efficiency in this example so that the relative CSU could be reduced. In this example it is assumed to be 5%.
The MQC can be calculated as:

$$y_Q = \frac{k_Q^2}{2 t_S \varepsilon (1 - k_Q^2 \phi_{\hat{\varepsilon}}^2)} \left(1 + \sqrt{1 + \frac{4(1 - k_Q^2 \phi_{\hat{\varepsilon}}^2)}{k_Q^2} \left(R_B t_S \left(1 + \frac{t_S}{t_B} \right) + 0 \right)} \right)$$

$$y_Q = \frac{100}{2\,(3000)(0.42)(0.75)} \left(1 + \sqrt{1 + \frac{4\,(0.75)}{100} \left((0.018\ \text{s}^{-1})(3000\ \text{s}) \left(1 + \frac{(3000\ \text{s})}{(6000\ \text{s})} \right) + 0 \right)} \right)$$

$= 0.151$ Bq/cm^2

As a check, y_Q can be calculated in a different way. If y_Q is the MQC and $k_Q = 10$, then the relative CSU of a measurement of concentration y_Q is 10%. The procedure described in Section 7.4 can be used to predict the CSU of a measurement made on a hypothetical sample whose concentration is exactly $y_Q = 0.151$ Bq/cm^2.

The measurement model is $Y = \frac{N_S - (N_B t_S / t_B)}{t_S \varepsilon}$.

Recall from Section 7.8.1.6 that if $y = \frac{f(x_1, x_2, \ldots, x_n)}{z_1 z_2 \ldots z_m}$, where f is some specified function of $x_1, x_2, \ldots, x_n$, all the z_i are nonzero, and all the input estimates are uncorrelated that the CSU may be calculated using Equation 7-35:

$$u_c^2(y) = \frac{u_c^2(f(x_1, x_2, \ldots, x_n))}{z_1 z_2 \ldots z_m} + y^2 \left(\frac{u^2(z_1)}{z_1^2} + \frac{u^2(z_2)}{z_2^2} + \ldots + \frac{u^2(z_m)}{z_m^2} \right)$$

Substituting
$y = Y$,
$f(x_1, x_2, \ldots, x_n) = f(N_S, N_B, t_S, t_B) = N_S - (N_B t_S / t_B) / t_S$,
$z_1 = \varepsilon$, and

$$u_c^2(N_S - (N_B t_S / t_B) / t_S) = u_c^2(N_S / t_S) + u_c^2((N_B t_S / t_B) / t_S) = \frac{u_c^2(N_S) + (t_S / t_B)^2 u_c^2(N_B)}{t_S^2} =$$

$$\frac{\sqrt{N_S}^2 + \sqrt{N_B}^2 (t_S^2 / t_B^2)}{t_S^2} = \frac{N_S + N_B (t_S^2 / t_B^2)}{t_S^2}$$

Results in:

$$u_c^2(Y) = \frac{N_S + (N_B t_S^2 / t_B^2)}{t_S^2 \varepsilon^2} + Y^2 \left(\frac{u^2(\varepsilon)}{\varepsilon^2} \right) \text{ or}$$

$$u_c(Y) = \sqrt{\frac{N_S + (N_B t_S^2 / t_B^2)}{t_S^2 \varepsilon^2} + Y^2 \left(\frac{u^2(\varepsilon)}{\varepsilon^2} \right)}$$

Inserting the values

$Y = y_Q = 0.151$ Bq/cm^2

$t_B = 6{,}000$ s

$t_S = 3{,}000$ s

$\varepsilon = 0.42$ (counts per second)/(Bq/cm^2)

$N_B = R_B t_B = (0.018\,\text{s}^{-1})(3{,}000\,\text{s}) = 108$ and

$N_S = x_Q t_S \varepsilon + R_B t_B = (0.151\text{ Bq})(3000\text{ s})(0.42) + (0.018\text{ s}^{-1})(3{,}000\text{ s}) = 244.26$

yields

$$u_c(Y) = \sqrt{\frac{244.26 + (108)(3{,}000)^2/(6{,}000)^2}{(3000)^2(0.42)^2} + (0.151)^2(0.05^2)} = 0.0151 \text{ Bq/cm}^2$$

Thus, the uncertainty at $y_Q = 0.151$ is 0.0151 and the relative uncertainty is 0.1, so y_Q is verified to be the MQC.

As above in this example, we adjust for the (now) 5% relative CSU in the counting efficiency. The uncertainty is (0.05) × (0.42) = 0.02142. Assuming that the efficiency is normally distributed, the lower 5th percentile is (0.42) - (1.645)(0.021) = 0.385. Therefore a conservative estimate of the efficiency is $\varepsilon = 0.385$ and a conservative estimate of the minimum detectable concentration is: $y_Q = \frac{(0.151)(0.42)}{0.385} = 0.165$ Bq/cm^2.

7.11 Calculate Scan MDCs

The methodology used to determine the scan MDC is based on NUREG-1507 (NRC 1998b). This procedure is quite complex as it requires, among other skills, a familiarity with radiation transport calculations for its implementation. The information developed here will be used in the example in Section 8.2, "Mineral Processing Facility Concrete Rubble." However, the details given in this section are not essential to understanding the example.

The radionuclides of concern are the members of the natural uranium and thorium series. The instrument used is a "Field Instrument for the Detection of Low Energy Radiation" (FIDLER). The approach used would be similar for other instruments and radionuclides.

The approach to determine scan MDCs includes:

- Calculate the fluence rate relative to the exposure rate (FRER) for the range of energies of interest (Section 7.11.1).
- Calculate the probability of interaction (*P*) between the radiation of interest and the detector (Section 7.11.2).
- Calculate the relative detector response (RDR) for each of the energies of interest (Section 7.11.3).
- Determine the relationship between the detector's net count rate to net exposure rate in cpm/μR/h, Section 7.11.4).
- Determine the relationship between the detector response and the radionuclide concentration (Section 7.11.5).
- Obtain the minimum detectable count rate (MDCR) for the ideal observer, for a given level of performance, by postulating detector background and a scan rate or observation interval (Section 7.11.6).
- Relate the MDCR for the ideal observer to a radionuclide concentration (in Bq/kg) to calculate the scan MDC (Section 7.11.7).

7.11.1 Calculate the Relative Fluence Rate to Exposure Rate (FRER)

For particular gamma energies, the relationship of NaI scintillation detector count rate and exposure rate may be determined analytically (in cpm/μR/h). The approach is to determine the gamma fluence rate necessary to yield a fixed exposure rate (μR/h) as a function of gamma energy. The fluence rate, following NUREG-1507 (NRC 1998b), is directly proportional to the exposure rate and inversely proportional to the incident photon energy and mass energy absorption coefficient:

$$\textit{Fluence Rate}(\text{FRER}) \propto \dot{X}\,\frac{1}{E_\gamma}\,\frac{1}{(\mu_{en}/\rho)_{air}} \tag{7-61}$$

Where:

$\dot{X}$ = exposure rate (set equal to 1 μR/hr for these calculations)
E_γ = energy of the gamma photon of concern (keV)
$(\mu_{en}/\rho)_{air}$ = mass energy absorption coefficient in air at the gamma photon energy of concern (cm^2/g)

The mass energy absorption coefficients in air are presented in Table 7.8 (natural uranium) and Table 7.9 (natural thorium) along with the calculated fluence rates (up to a constant of proportionality, since only the ratios of these values are used in subsequent calculations). Note that while the mass energy absorption coefficients in air, $(\mu_{en}/\rho)_{air}$, are tabulated values (NIST 1996), the selected energies are determined by the calculation of the detector response based on radionuclide concentration (Section 7.11.5).

7.11.2 Calculate the Probability of Interaction

Assuming that the primary gamma interaction producing the detector response occurs through the end of the detector (i.e., through the beryllium window of the detector, as opposed to the sides), the probability of interaction (*P*) for a gamma may be calculated using Equation 7-52:

$$P = 1 - e^{-(\mu/\rho)_{NaI}(x)(\rho_{NaI})} = 1 - e^{-(0.117\ cm^2/g)(0.16\ cm)(3.67\ g/cm^3)} = 0.066 \text{ at } 400 \text{ keV} \quad (7\text{-}62)$$

Where:

P = probability of interaction (unitless)

$(\mu/\rho)_{NaI}$ = mass attenuation coefficient of FIDLER NaI crystal at the energy of interest (e.g., 0.117 cm^2/g at 400 keV)

x = thickness of the thin edge of the FIDLER NaI crystal (0.16 cm)

ρ = density of the NaI crystal (3.67 g/cm^3)

The mass attenuation coefficients for the NaI crystal and the calculated probabilities for each of the energies of interest are presented in Table 7.8 (natural uranium) and Table 7.9 (natural thorium). The mass attenuation coefficients for NaI were calculated using the XCOM program (NIST 1998).

Table 7.8 Calculation of Detector Response to Natural Uranium

Energy (keV)	$(\mu_{en}/\rho)_{air}$ (cm^2/g)	FRER (Section 7.11.1)	$(\mu/\rho)_{NaI}$ cm^2/g	*P* (Section 7.11.2)	RDR (Section 7.11.3)	cpm per µR/h (Section 7.11.4)
15	1.334	0.04998	47.4	1.000	0.04998	28,374
20	0.5389	0.09278	21.8	1.000	0.09278	52,678
30	0.1537	0.2169	7.36	0.9867	0.2140	121,498
40	0.06833	0.3659	18.8	1.000	0.3659	207,725
50	0.04098	0.4880	10.5	0.9979	0.4870	276,511
60	0.03041	0.5481	6.45	0.9773	0.5356	304,123
80	0.02407	0.5193	3.00	0.8282	0.4301	244,204
100	0.02325	0.4301	1.67	0.6249	0.2688	152,606
150	0.02496	0.2671	0.611	0.3015	0.08052	45,717
200	0.02672	0.1871	0.328	0.1752	0.03278	18,613
300	0.02872	0.1161	0.166	0.09288	0.01078	6,120
400	0.02949	0.08477	0.117	0.06640	0.005629	3,196
500	0.02966	0.06743	0.0950	0.05426	0.003659	2,077
600	0.02953	0.05644	0.0822	0.04712	0.002660	1,510
662	0.02931	0.05154	0.0766	0.04398	0.002267	1,287
800	0.02882	0.04337	0.0675	0.03886	0.001685	957
1,000	0.02789	0.03586	0.0588	0.03394	0.001217	691
1,500	0.02547	0.02617	0.0470	0.02722	0.0007125	405
2,000	0.02345	0.02132	0.0415	0.02407	0.0005133	291

Table 7.9 Calculation of Detector Response for Natural Thorium

Energy (keV)	$(\mu_{en}/\rho)_{air}$ (cm^2/g)	FRER (Section 7.11.1)	$(\mu/\rho)_{NaI}$ cm^2/g	*P* (Section 7.11.2)	RDR (Section 7.11.3)	cpm per µR/h (Section 7.11.4)
40	0.06833	0.3659	18.8	1.000	0.3659	207,725
60	0.03041	0.5481	6.45	0.9773	0.5356	304,123
80	0.02407	0.5193	3.00	0.8282	0.4301	244,204
100	0.02325	0.4301	1.67	0.6249	0.2688	152,606
150	0.02496	0.2671	0.611	0.3015	0.08052	45,717
200	0.02672	0.1871	0.328	0.1752	0.03278	18,613
300	0.02872	0.1161	0.166	0.09288	0.01078	6,120
400	0.02949	0.08477	0.117	0.06640	0.005629	3,196
500	0.02966	0.06743	0.0950	0.05426	0.003659	2,077
600	0.02953	0.05644	0.0822	0.04712	0.002660	1,510
662	0.02931	0.05154	0.0766	0.04398	0.002267	1,287
800	0.02882	0.04337	0.0675	0.03886	0.001685	957
1,000	0.02789	0.03586	0.0588	0.03394	0.001217	691
1,500	0.02547	0.02617	0.0470	0.02722	0.0007125	405
2,000	0.02343	0.02134	0.0415	0.02407	0.0005137	292
3,000	0.02057	0.01620	0.0368	0.02138	0.0003464	197

7.11.3 Calculate the Relative Detector Response

The relative detector response (RDR) for each of the energies of interest is determined by multiplying the FRER by *P*. The results are presented in Table 7.8 (natural uranium) and Table 7.9 (natural thorium).

7.11.4 Relationship Between Detector Response and Exposure Rate

Using the same methodology described in Sections 7.11.1 through 7.11.3, FRER, P, and RDR are calculated at the ^{137}Cs energy of 662 keV and are also presented in Table 7.8 and Table 7.9. The manufacturer of the FIDLER NaI detector provides an estimated response of the crystal in a known radiation field, which is 1,287 cpm per µR/h at the ^{137}Cs energy of 662 keV. The response at 662 keV can be used to determine the response at all other energies of interest using Equation 7-63:

$$\frac{\text{cpm}}{\mu\text{R/h}_{E_i}} = \left(\frac{1{,}287\ \text{cpm}}{\mu\text{R/h}}\right) \times \frac{\text{RDR}_{E_i}}{\text{RDR}_{^{137}\text{Cs}}} \qquad (7\text{-}63)$$

Where:

E_i = energy of the photon of interest (keV)

$\frac{\text{cpm}}{\mu\text{R/h}_{E_i}}$ = response of the detector for energies of interest, Table 7.8 and Table 7.9

RDR_{E_i} = RDR at the energy of interest, Table 7.8 and Table 7.9

$\text{RDR}_{^{137}\text{Cs}}$ = RDR for ^{137}Cs, Table 7.8 and Table 7.9

The responses in cpm per μR/h for each of the decay energies of interest are presented in Tables 7.8 and 7.9.

7.11.5 Relationship Between Detector Response and Radionuclide Concentration

The minimum detectable exposure rate is used to determine the MDC by modeling a specific impacted area. The relationship between the detector response (in cpm) and the radionuclide concentration (in Bq/kg) uses a computer gamma dose modeling code to model the presence of a normalized 1 Bq/kg total activity source term for natural uranium and natural thorium. The following assumptions from NUREG-1507 (NRC 1998b) were used to generate the computer gamma dose modeling runs:

- Impacted media is concrete,
- Density of concrete is 2.3 g/cm^3,
- Activity is uniformly distributed into a layer of crushed concrete 15 cm thick,
- Measurement points are 10 cm above the concrete surface,
- Areas of elevated activity are circular with an area of 0.25 m^2 and a radius of 28 cm,
- 0.051 cm beryllium shield simulates the window of the FIDLER detector, and
- Normalized 1 Bq/kg source term decayed for 50 years to allow ingrowth of decay progeny.

The weighted cpm per μR/h response (weighted instrument sensitivity [WS_i]) for each decay energy is calculated by multiplying the μR/h at 1 Bq/kg (exposure rate with buildup, R_i) by the cpm per μR/h and dividing by the total μR/h (at 1 Bq/kg) for all decay energies of interest (Equation 7-64):

$$WS_i = \frac{R_i \times (\text{cpm per } \mu\text{R/h})}{R_T} \quad (7\text{-}64)$$

Where:

WS_i = weighted instrument sensitivity (cpm per μR/h)
R_i = exposure rate with buildup (μR/h)
R_T = Total exposure rate with buildup (μR/h)

Calculate the percent of FIDLER response for each of the decay energies of interest by dividing WS_i by the total weighted cpm per μR/h and multiplying by 100 percent (Equation 7-62):

$$\text{Percent of FIDLER response} = \frac{WS_i \times 100\%}{W_T} \quad (7\text{-}65)$$

Where:

W_T = Total WS_i weighted instrument sensitivity (cpm per μR/h)

The exposure rates for each of the decay energies of interest are presented in Table 7.10 (assuming natural uranium for the source term) and Table 7.11 (assuming natural thorium for the source term).

Table 7.10 Detector Response to Natural Uranium

Energy keV	R_i (µR/h) (Section 7.11.5)	cpm per µR/h (Section 7.11.4)	WS_i (cpm per µR/h) (Section 7.11.5)	Percent of FIDLER Response (Section 7.11.5)
15	4.473×10^{-10}	28,374	0	0.00%
20	3.597×10^{-12}	52,678	0	0.00%
30	2.623×10^{-07}	121,498	226	0.504%
40	1.299×10^{-10}	207,725	0	0.00%
50	1.052×10^{-07}	276,511	206	0.460%
60	5.065×10^{-06}	304,123	10903	24.3%
80	1.518×10^{-06}	244,204	2625	5.86%
100	2.309×10^{-05}	152,606	24938	55.7%
150	5.138×10^{-06}	45,717	1663	3.71%
200	2.881×10^{-05}	18,613	3796	8.48%
300	2.237×10^{-07}	6,120	10	0.0216%
400	2.434×10^{-07}	3,196	6	0.0123%
500	4.208×10^{-07}	2,077	6	0.0138%
600	2.048×10^{-06}	1,510	22	0.0489%
800	1.478×10^{-05}	957	100	0.224%
1,000	5.759×10^{-05}	691	282	0.629%
1,500	1.695×10^{-06}	405	5	0.0108%
2,000	2.841×10^{-07}	291	1	0.00131%
Total	1.413×10^{-04}		44,923	100%

Table 7.11 Detector Response to Natural Thorium

Energy keV	R_i (µR/h) (Section 7.11.5)	cpm per µR/h (Section 7.11.4)	WS_i (cpm per µR/h) (Section 7.11.5)	Percent of FIDLER Response (Section 7.11.5)
40	1.299×10^{-06}	207,725	10	0.266%
60	1.816×10^{-06}	304,123	21	0.544%
80	1.989×10^{-04}	244,204	1855	47.8%
100	5.027×10^{-05}	152,606	293	7.55%
150	5.862×10^{-05}	45,717	102	2.64%
200	1.135×10^{-03}	18,613	807	20.8%
300	8.922×10^{-04}	6,120	209	5.37%

Table 7.11 Detector Response to Natural Thorium (Continued)

Energy keV	R_i (µR/h) (Section 7.11.5)	cpm per µR/h (Section 7.11.4)	WS_i (cpm per µR/h) (Section 7.11.5)	Percent of FIDLER Response (Section 7.11.5)
400	1.105×10^{-04}	3,196	13	0.348%
500	8.146×10^{-04}	2,077	65	1.67%
600	2.218×10^{-03}	1,510	128	3.30%
800	2.892×10^{-03}	957	106	2.72%
1,000	6.443×10^{-03}	691	170	4.38%
1,500	2.062×10^{-03}	405	32	0.821%
2,000	5.822×10^{-05}	292	1	0.0167%
3,000	9.249×10^{-03}	197	69	1.79%
Total	2.619×10^{-02}		3881	100%

7.11.6 Calculation of Scan Minimum Detectable Count Rates

In the computer gamma dose modeling, an impacted area with a radius of 28 cm or approximately 0.25 m was assumed. Using a scan speed of 0.25 m/s provides an observation interval of one second.

A typical background exposure rate is 10 μR/h. Using a conversion factor based upon field measurements of 1,287 cpm per μR/h for ^{137}Cs (see 7.11.4) results in an estimated background count rate of 12,870 cpm. Converting this value from cpm to counts per second (cps) using Equation 7-66 results in a background of 214.5 cps.

$$b(\text{cpm})\times\frac{1\,\text{min}}{60\,\text{sec}}\times i(\text{sec})=\frac{1,287\ \text{cpm}}{1\ \mu\text{R/h}}\times 10\ \mu\text{R/h}\times\frac{1\,\text{min}}{60\,\text{sec}}\times 1\,\text{sec}=214.5\ \text{cps} \qquad (7\text{-}66)$$

Where:

b = background count rate (12,870 cpm)
i = observation interval length (1 s)

The MDCR is calculated using the methodology in NUREG-1507 (NRC 1998b) shown in Equations 7-67 and 7-68:

$$s_i = d'\sqrt{b_i} = 1.38\times\sqrt{214.5} = 20.21\ \text{counts} \qquad (7\text{-}67)$$

$$s_{i,\,surveyor} = \frac{d'\sqrt{b_i}}{\sqrt{p}} = \frac{1.38\times\sqrt{214.5}}{\sqrt{0.5}} = 28.58\ \text{counts}$$

$$\text{MDCR} = s_i\times(60/i) = 20.21\times(60/1) = 1,212\ \text{cpm} \qquad (7\text{-}68)$$

$$\text{MDCR}_{surveyor} = s_{i,\,surveyor} \times (60/i) = 28.58 \times (60/1) = 1{,}715 \text{ cpm}$$

Where:

b_i	=	average number of counts in the background interval (214.5 counts)
i	=	observation interval length (one second)
p	=	efficiency of a less than ideal surveyor, range of 0.5 to 0.75 from NUREG-1507 (NRC 1998b); a value 0.5 was chosen as a conservative value
d'	=	detectability index from Table 6.1 of NUREG-1507 (NRC 1998b); a value of 1.38 was selected, which represents a true positive detection rate of 95% and a false positive detection rate of 60%[12]
s_i	=	minimum detectable number of net source counts in the observation interval (counts)
$s_{i,surveyor}$	=	minimum detectable number of net source counts in the observation interval by a less than ideal surveyor
MDCR	=	minimum detectable count rate (cpm)
$\text{MDCR}_{surveyor}$	=	MDCR by a less than ideal surveyor (cpm)

7.11.7 Calculate the Scan Minimum Detectable Concentration

The scan minimum detectable concentration (MDC) can be calculated from the minimum detectable exposure rate (MDER). The MDER can be calculated using the previously calculated total weighted instrument sensitivities (WS_i), in cpm per μR/h, for natural uranium and natural thorium as shown in Equations 7-69 and 7-70:

$$\text{MDER} = \frac{\text{MDCR}_{surveyor}}{W_T} \tag{7-69}$$

$$\text{Scan MDC} = C \times \frac{\text{MDER}}{R_T} \tag{7-70}$$

Where:

MDER	=	MDER for the "ith" source term, by a less than ideal surveyor, (μR/h)
$\text{MDCR}_{surveyor}$	=	MDCR rate by a less than ideal surveyor (cpm), from Section 7.11.6
W_T	=	Total weighted instrument sensitivity (cpm per μR/h, Table 7.10 and Table 7.11)
R_T	=	Total exposure rate with buildup (μR/h, Table 7.10 and Table 7.11)
C	=	concentration of source term (set at 1 Bq/kg in Section 7.11.5)

[12] A Type I error, misidentifying a background area as elevated will have the consequence that a longer reading will be needed to verify the initial decision. This will happen with probability α. A Type II error, missing a true elevated area, may lead to incorrectly exceeding the limit for the chosen disposition option. This will happen with probability β. Since in this instance the consequences of a Type I error are often considered much lower than the consequences associated with a Type II error. Thus, α may be set higher than β. Setting both very low could result in slow scanning speeds and operator fatigue.

Scan MDC = minimum detectable concentration (Bq/kg)

The Scan MDCs for the FIDLER were calculated using Equations 7-69 and 7-70 and the instrument response information from Table 7.10 (assuming natural uranium as the source term) and Table 7.11 (assuming natural thorium as the source term). The scan MDCs for natural uranium and natural thorium using a FIDLER are listed in Table 7.12.

Table 7.12 Scan MDCs for FIDLER

Source Term	$MDCR_{surveyor}$ (cpm) Section 7.11.6	W_T (cpm per µR/h) Section 7.11.5	MDER (µR/h) Section 7.11.7	R_T (µR/h) Section 7.11.5	C (Bq/kg) Section 7.11.5	Scan MDC (Bq/kg) Section 7.11.7
Natural Uranium	1,715	44,786	0.03829	1.413×10^{-04}	1	**271 ≈ 300**
Natural Thorium	1,715	3,881	0.4419	2.619×10^{-02}	1	**16.9 ≈ 20**

The scan MDCs of approximately 300 Bq/kg for uranium and 20 Bq/kg for thorium are both less than their respective action levels of 38,000 and 330 Bq/kg, respectively.