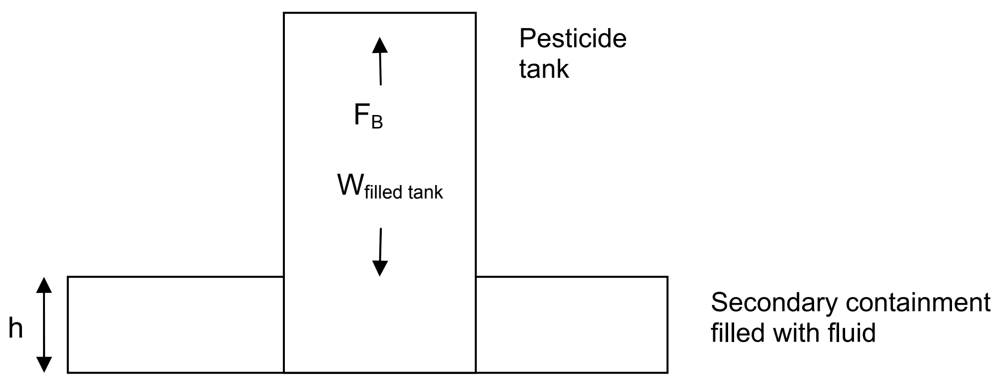
Buoyant Force Calculations

When a rigid object is submerged in a fluid (completely or partially), there exists an upward force on the object that is equal to the weight of the fluid that is displaced by the object. In terms of the situation of a tank in a secondary containment unit filled with liquid, this means that a tank will not float if the weight of the tank (including any pesticide in it) is greater than the buoyant force pushing up. The strength of the buoyant force depends on the density of the fluid that could fill the secondary containment (e.g., water, pesticide,or fertilizer) and the volume of the tank that is below the secondary containment wall.

Assume that there is a cylindrical, flat-bottom tank that sits on the floor of a secondary containment unit. The secondary containment is filled with a fluid (water, pesticide, or fertilizer) that has a density of ρ_{fluid} (in pounds/gallon) and the height of the secondary containment wall is h (in feet).



The **weight of the filled tank ($W_{filled\ tank}$)** is equal to the weight of the empty tank plus the weight of the pesticide in the tank (which is the volume of the pesticide in the tank multiplied by the density of the pesticide):

$W_{filled\ tank} = W_{empty\ tank} + W_{pesticide}$

$W_{filled\ tank} = mass_{empty\ tank} * g + \rho_{pesticide} * Volume_{pesticide} * g$, where g = acceleration due to gravity.

The **buoyant force (F_B)** is equal to the weight of the fluid displaced by the tank.

F_B = weight fluid that is displaced = $mass_{fluid} * g$

$mass_{fluid}$ = density of fluid * volume of tank that is submerged

$F_B = \rho_{fluid} * Volume_{tank\ submerged} * g$

For the tank to not float, the weight of the filled tank must be greater than the buoyant force:

$W_{filled\ tank} > F_B$

$W_{empty\ tank} + W_{pesticide} > F_B$

$mass_{empty\ tank} * g + \rho_{pesticide} * Volume_{pesticide} * g > \rho_{fluid} * Volume_{tank\ submerged} * g$

(Dividing this equation by “g” gives the following:)

$\mathbf{mass_{empty\ tank} + \rho_{pesticide} * Volume_{pesticide} > \rho_{fluid} * Volume_{tank\ submerged}}$

This equation really boils down to: the mass of the filled tank (the tank itself and the pesticide in it) must be greater than the mass of the fluid displaced by the tank. Fortunately, the parts of this equation are all things that will be known for a given situation: (1) the mass of the empty tank; (2) the density of the pesticide, (3) the volume of pesticide in the tank, (4) the density of the fluid that fills the secondary containment (the calculations should be done using the most dense fluid that could fill the secondary containment), and (5) the volume of the tank that is submerged.

One other equation is needed to calculate the volume in a cylindrical tank:

Volume of a cylinder (in gallons) = $d^2 * 5.874 * h$, where

d = the tank diameter (in feet) and
h = the height of the cylinder (in feet), i.e., the secondary containment wall height when calculating the volume of the tank that is submerged or the level of pesticide in the tank when calculating the mass of the pesticide in the tank.

Here are several examples of the calculations. All three examples use the same tank size and secondary containment height. However, the minimum required height of pesticide in the tank and the material that fills the secondary containment unit (fertilizer or pesticide) varies. The situation is:

- A 2,500-gallon cylindrical steel tank is in secondary containment. The mass of the tank is 1,235 pounds and the tank’s diameter is 6 feet.
- The wall of the secondary containment unit is 2 feet high.
- The tank holds pesticide with a density of 8.5 pounds/gallon. (The density of water is 8.3 pounds/gallon.)

Example 1: Pesticide and fertilizer are in the same secondary containment unit, with a minimum of 4 feet of pesticide in the tank.

The secondary containment unit also includes a tank that holds liquid fertilizer with a density of 11.6 pounds/gallon. The facility has set the minimum level of pesticide in the tank to be 4 feet.

The worst case (biggest buoyant force) would be if the fertilizer tank failed and the secondary containment unit filled with liquid fertilizer. In this case, the mass of liquid fertilizer displaced by the tank (essentially the buoyant force) would be:

Mass of fluid displaced by tank = $\rho_{fertilizer} * Volume_{tank\ submerged}$

First, calculate the volume of the tank that is submerged in the secondary containment unit:

$Volume_{tank\ submerged}$ (in gal) = $d^2 * 5.874 * h$ = 6 ft * 6 ft * 5.874 * 2 ft = 423 gal.

Then calculate the mass of fertilizer displaced by the tank, which is 4,907 pounds:

Mass of fluid displaced by tank = $\rho_{fertilizer}$ * $Volume_{tank\ submerged}$ = 11.6 lb/gal * 423 gal = 4,907 lb

To determine the mass of the filled tank, first calculate the volume of pesticide in the tank:

$Volume_{pesticide\ in\ tank}$ (in gal) = d^2 * 5.874 * h = 6 ft * 6 ft * 5.874 * 4 ft = 846 gal.

Then determine the mass of the filled tank, which is 8,426 pounds:

$mass_{filled\ \ tank}$ = $mass_{empty\ tank}$ + $\rho_{pesticide}$ * $volume_{pesticide}$ = 1,235 lb + 8.5 lb/gal * 846 gal
$mass_{filled\ \ tank}$ = 8,426 lb.

Because the mass of the filled tank (8,426 pounds) is greater than the mass of fluid displaced by the tank (4,907 pounds), the tank would be considered to be anchored and would not float if the secondary containment unit filled with liquid fertilizer, that pesticide, or water. (The pesticide and water are less dense than the fertilizer, so the buoyant force would be smaller.)

Example 2: Pesticide and fertilizer are in the same secondary containment unit, with a minimum of 2 feet of pesticide in the tank.

The secondary containment unit also has a tank that holds liquid fertilizer with a density of 11.6 pounds/gallon. The facility wants to set the minimum level of pesticide in the tank to be 2 feet.

The mass of the fertilizer displaced by the tank is the same as in Example 1 and is 4,907 pounds.

The mass of the filled tank is different than in Example 1, because there is a smaller amount of pesticide in the tank:

$Volume_{pesticide\ in\ tank}$ (in gal) = d^2 * 5.874 * h = 6 ft * 6 ft * 5.874 * 2 ft = 423 gallons

$mass_{filled\ \ tank}$ = $mass_{empty\ tank}$ + $\rho_{pesticide}$ * $Volume_{pesticide}$ = 1,235 lb + 8.5 lb/gal * 423 gal
$mass_{filled\ \ tank}$ = 4,831 lb

In this example, the mass of the filled tank (4,831 pounds) is less than the mass of fluid displaced by the tank (4,907 pounds), so the tank would not be considered to be anchored and would float if the secondary containment filled with liquid fertilizer.

Example 3: Only pesticide is in the secondary containment unit, with a minimum of 2 feet of pesticide in the tank.

Liquid fertilizer is in a separate secondary containment unit, and the pesticide with a density of 8.5 pounds/gallon is the most dense material stored in the secondary containment unit. The facility wants to set the minimum level of pesticide in the tank to be 2 feet.

Because this pesticide is more dense than water, the worst case (biggest buoyant force) would be if the secondary containment unit filled with this pesticide. In this case, the mass of fluid displaced by the tank would be:

Mass of fluid displaced by tank = $\rho_{pesticide}$ * $Volume_{tank\ submerged}$ = 8.5 lb/gal * 423 gal = 3,596 lb

The mass of the filled tank is 4,831 pounds, as calculated in Example 2:

$$mass_{filled\ tank} = mass_{empty\ tank} + \rho_{pesticide} * Volume_{pesticide} = 4{,}831\ lb$$

In this example, the mass of the filled tank (4,831 pounds) is greater than the mass of the fluid displaced by the tank 3,596 pounds), so the tank would be considered to be anchored and would not float if the secondary containment unit filled with that pesticide or water.

In other words, two feet of pesticide in the tank is enough to anchor it if the pesticide is the most dense fluid that could fill the secondary containment unit (in Example 3). However, two feet of pesticide in the tank is not enough to anchor it if the secondary containment unit also has a tank holding liquid fertilizer with a density of 11.6 pounds/gallon (in Example 2). These examples show that the specific conditions of each secondary containment unit must be considered to determine the level of pesticide in a tank that would make it heavy enough to be anchored for the purposes of complying with the containment regulations.