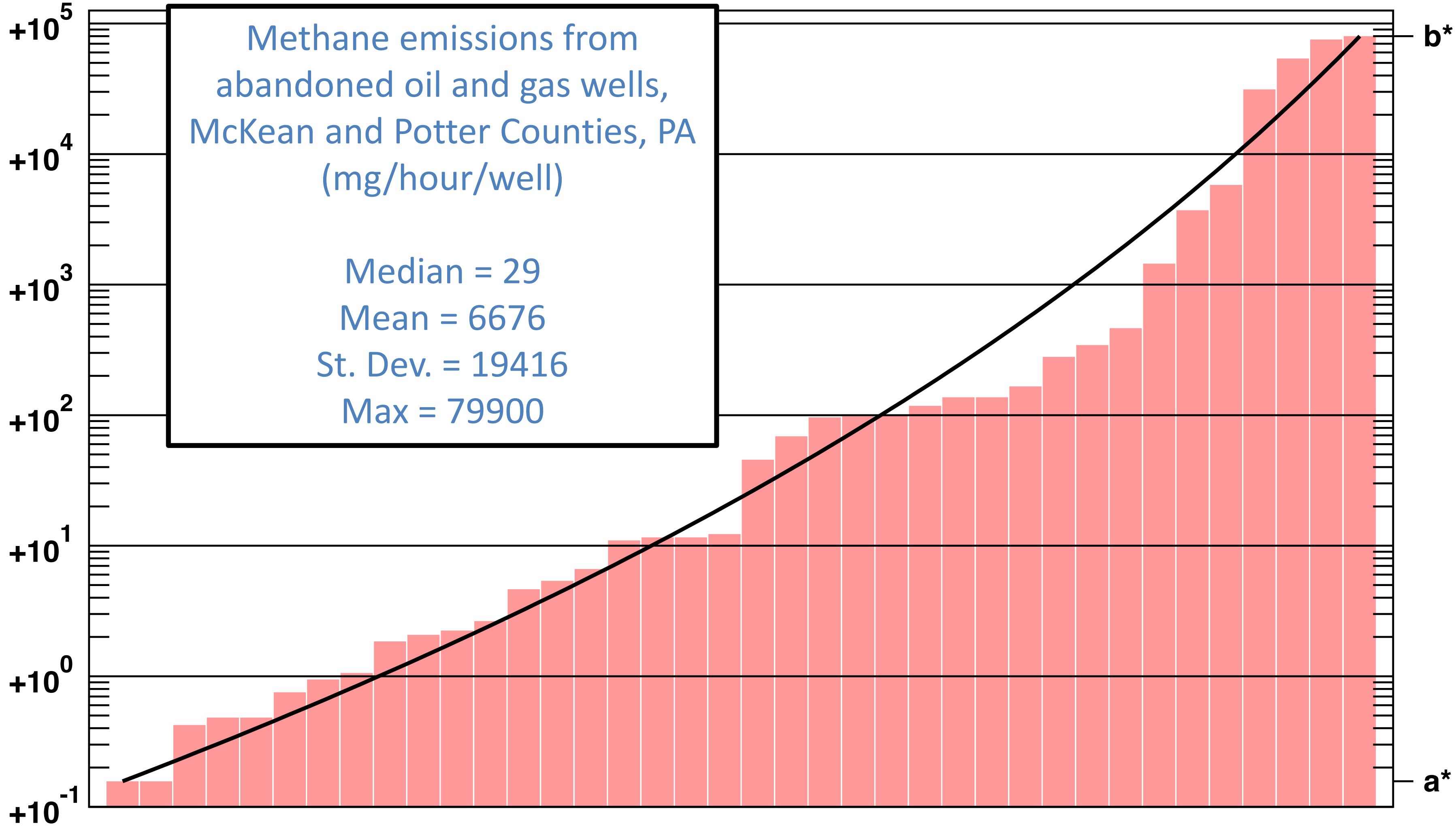
STATISTICS OF SUPER-EMITTERS: Modeling heavy-tailed datasets with power-law distributions

Marc Mansfield
Bingham Research Center
Utah State University
Vernal, Utah

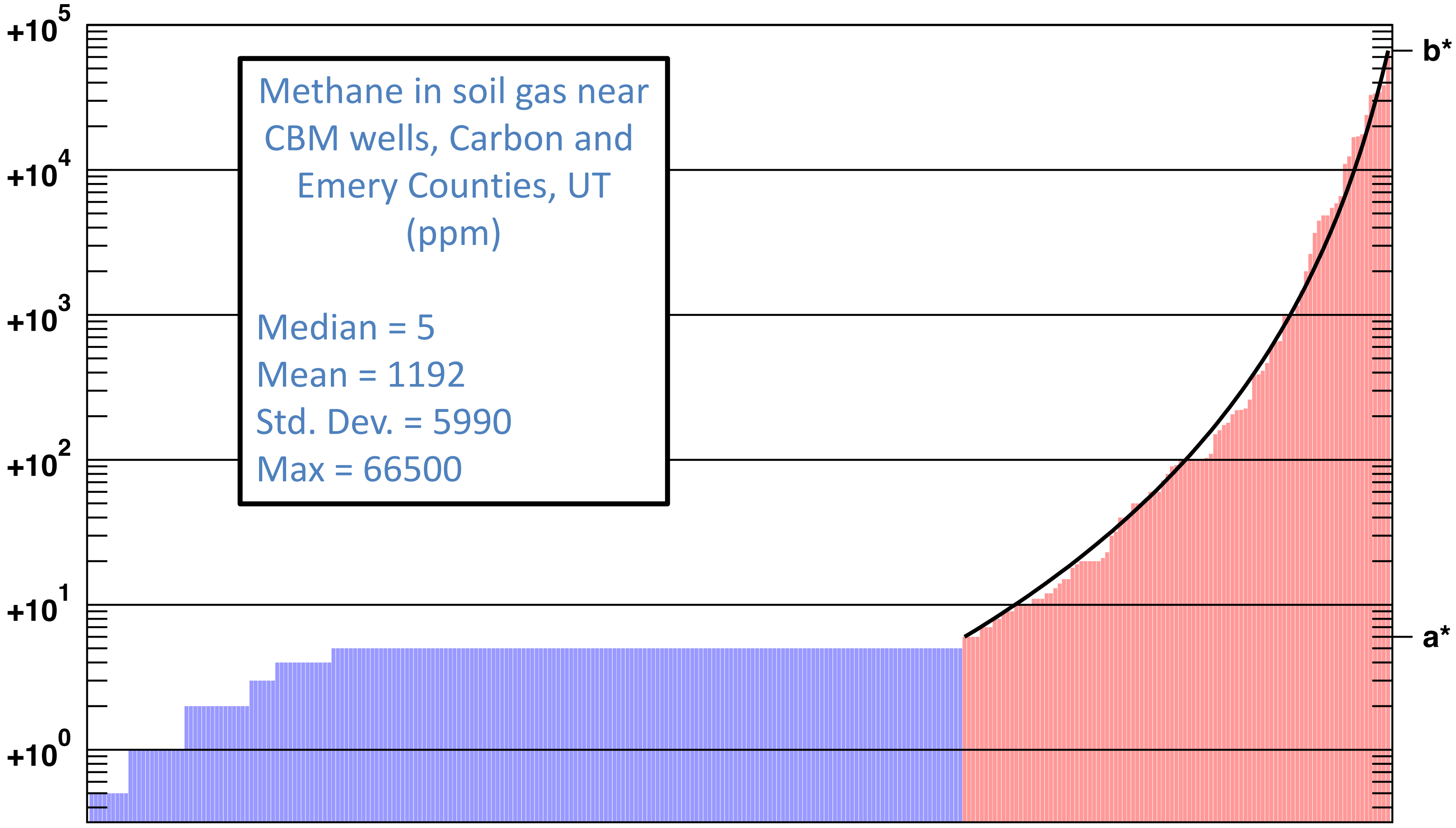
ACKNOWLEDGMENTS
Utah Science Technology and Research (USTAR) Initiative
Bureau of Land Management

COMMERCIALIZATION *and* REGIONAL DEVELOPMENT
UtahStateUniversity

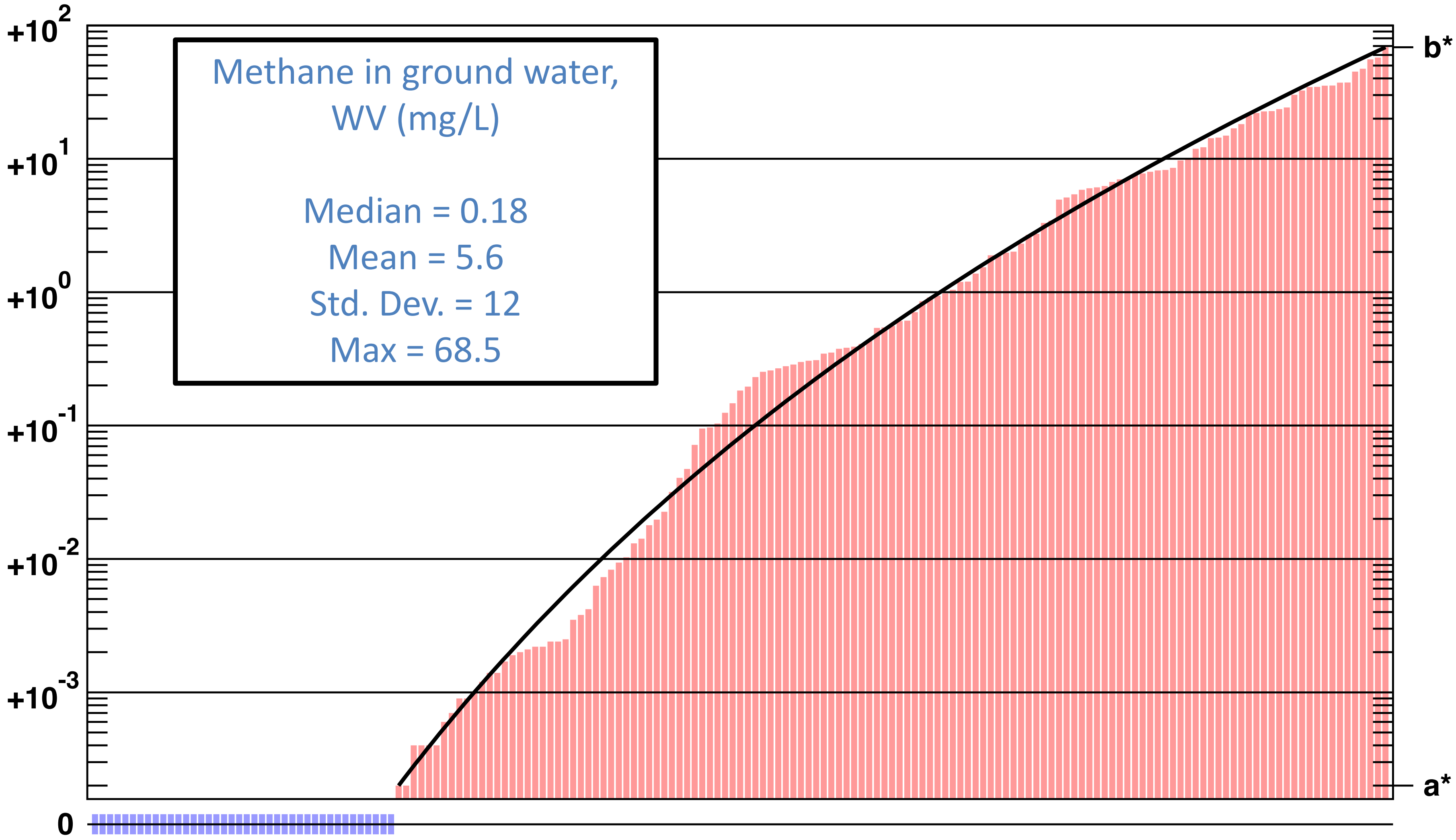
April 16, 2015
EPI Emissions Inventory Conference
San Diego



M. Kang, et al., “Direct measurements of methane emissions from abandoned oil and gas wells in Pennsylvania,” PNAS, 111, 18173-18177 (2014).



Stolp, Burr, and Johnson, "Methane Gas Concentration in Soils and Ground Water, Carbon and Emery Counties, Utah, 1995-2003," US Geological Survey, Scientific Investigations Report 2006-5227 (2006).



White and Mathes, "Dissolved-gas concentrations in ground water in West Virginia," U.S. Geological Survey Data Series 156 (2006).

Super-emitters:

High-end members of the dataset, “hot spots.”

Responsible for most of the emission.

(70%-30%, 80%-20% rules, etc.)

Distributions have “heavy” or “fat” tails:

Much of the weight of the distribution is in the tail.

Mean >> median

Have we adequately sampled the super-emitters?

Perhaps this explains growing suspicions than bottom-up inventories are too low.

The things you learned in Statistics 101 are of no help here.

Strategy to Analyze Heavy-Tailed Datasets
Step 1: Fit to a distribution

Fit dataset to a distribution, e.g., power-law.

$$P(x) = \frac{\beta}{x^{\lambda}}$$

Usually between upper and lower cutoffs: $a < x < b$

"Maximum Likelihood Estimation"

Upper cutoff is necessary whenever $\lambda < 2$.
(Earth can only produce a finite amount of methane.)

λ controls how rapidly the super-emitters thin out.

Why power laws?

<u>Generalized Central Limit Theorem:</u>

Gaussian distributions and power laws are "stable distributions."
Sums of large number of random variables: Gaussian
Sums of large number of heavy-tailed random variables: Power law
Products of large numbers of random variables: Log-normal

Long story short: Power laws are to heavy-tailed datasets what the Gaussian distribution is to run-of-the-mill datasets.

"One thus expects power laws to emerge naturally for rather unspecific reasons, simply as a by-product of mixing multiple (potentially rather disparate) heavy-tailed distributions." Stumpf & Porter, Science, 335, 666 (2012).

Like the Gaussian distribution, power-law distributions pop up everywhere:

- Personal wealth or income
- Species among genera
- Lunar craters
- Citations of scientific papers
- Stellar masses
- City sizes
- Files in internet traffic
- Occurrence frequency of words

Power Law Fits

(See also solid curves on bar charts.)

	l	r*	Range (max/min)
Pennsylvania Wells	1.08	0.68	500,000
Utah Soil Gas	1.21	0.77	11,000
West Virginia Ground Water	0.92	0.64	340,000

Indicates quality of fit

Strategy to Analyze Heavy-Tailed Datasets
Step 2: 95% confidence limits

"Based on the dataset in hand, we can state, with 95% confidence, that the true mean lies somewhere between A and B."

Fitted distribution ≠ "true" distribution

Many others are also good fits

Determine 95% confidence limits by averaging over all possible distributions.

This average is inherent in the formula they teach in Stat 101.

Not guaranteed to work for heavy-tailed sets.

95%-confidence algorithm for power law distributions works very well

IF

I know the upper cutoff, b.

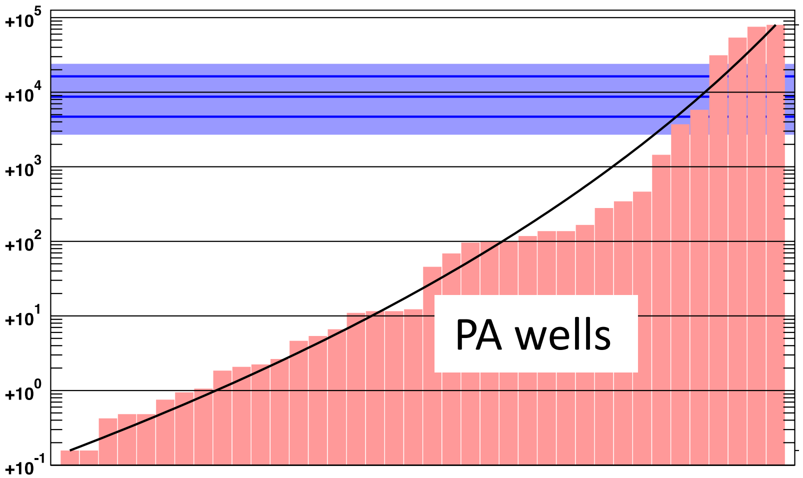
(Related to the infinities inherent in the power law.)

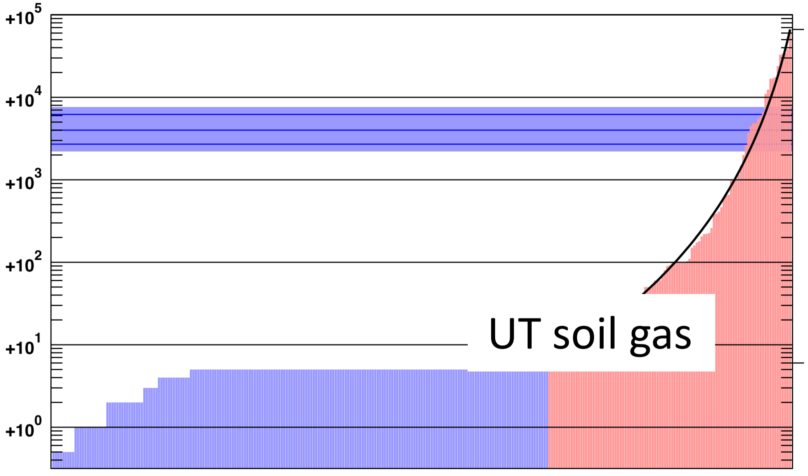
Sometimes we might have independent information:
e.g., methane in soil gas < 1,000,000 ppm
There may be other clues.
(I'm omitting the details.)

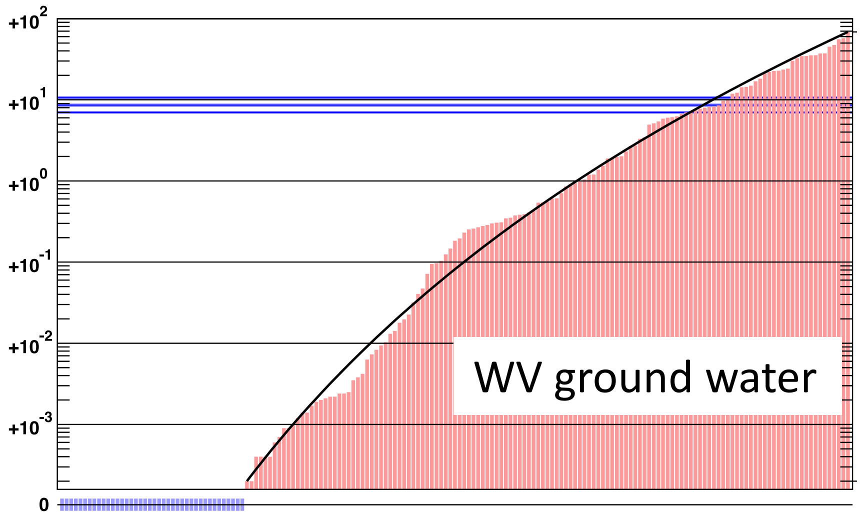
Without b, the 95%-confidence interval becomes blurred and fuzzy.

Large N helps.
$\lambda < 1$ or $\lambda > 3$ helps.

95% confidence limits (using best available procedure) become spread out and fuzzy.







I do not expect a similar problem for log-normal laws

BUT

which law is appropriate?

(It might be possible for the dataset itself to answer this question.)

Conclusions

- Many heavy-tailed datasets of environmental pollutants can be fit to power laws.

- 95%-confidence limit calculation often becomes "fuzzy." We can determine a confidence interval, but cannot always give it a definite percentage score. This is related to the inherent unpredictability of b.