Estimating Theoretical Error Distributions for Overflight Methane Measurements

Colorado School of Mines
Department of Applied Mathematics and Statistics

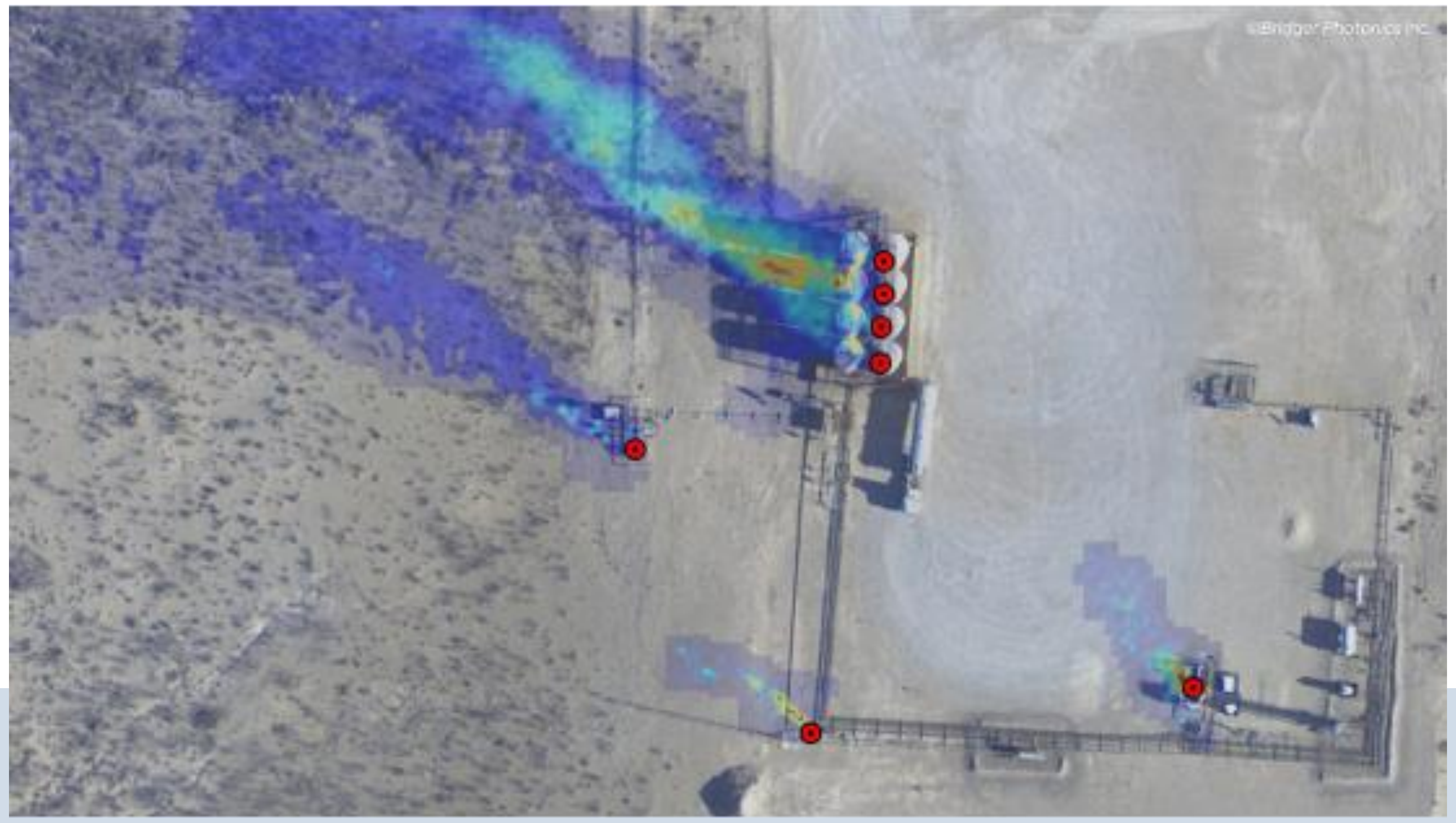
Daniel Callan Richards-Dinger
28 September 2023

COLORADO SCHOOL OF MINES
EARTH • ENERGY • ENVIRONMENT

Motivation

BRIDGER PHOTONICS

- Increased emphasis on measurement-informed inventories
- New technologies rapidly developing, e.g. Bridger Photonics
- Measurements far more useful with uncertainties
  - Need robust methods for estimating uncertainty

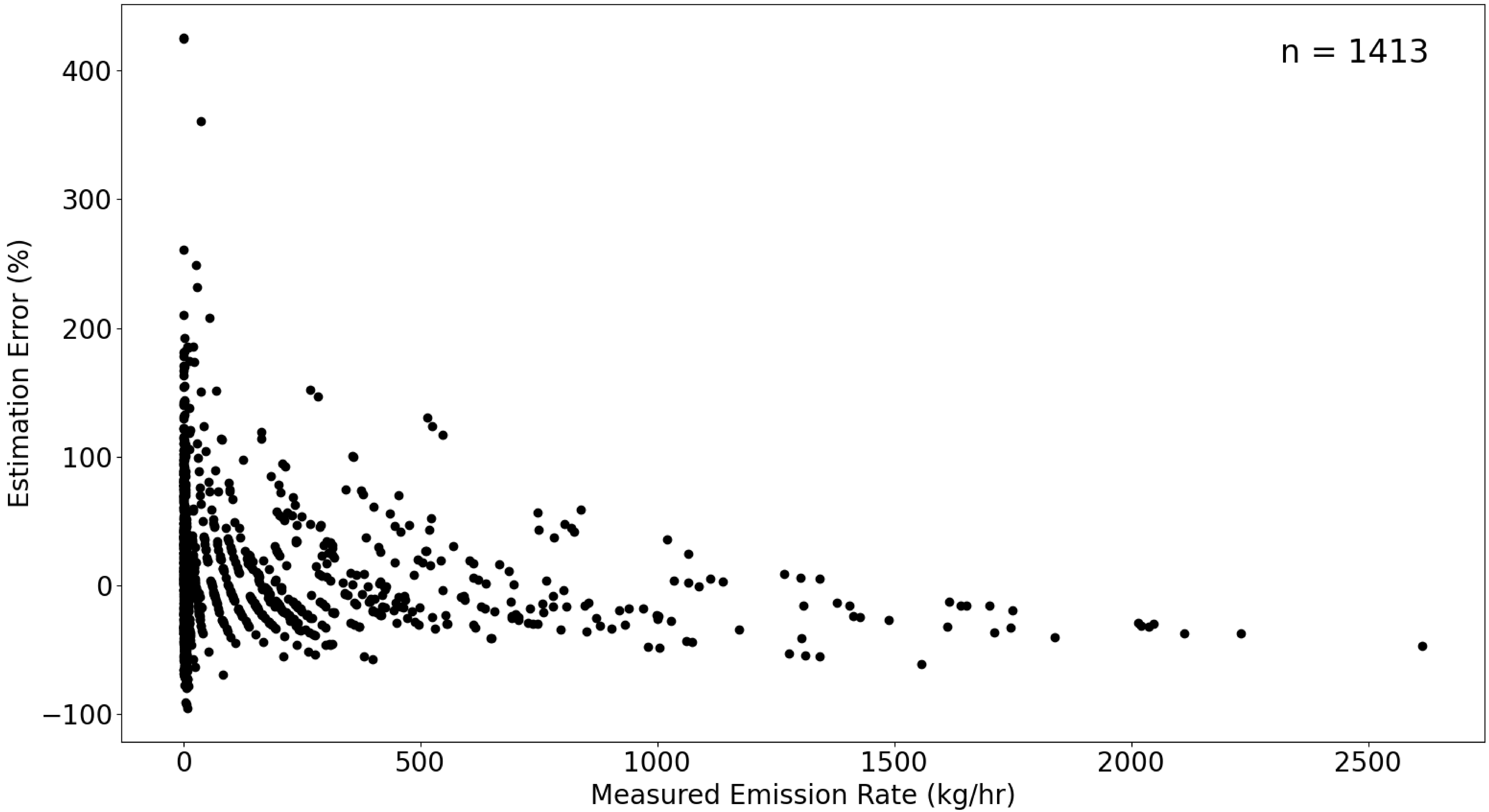


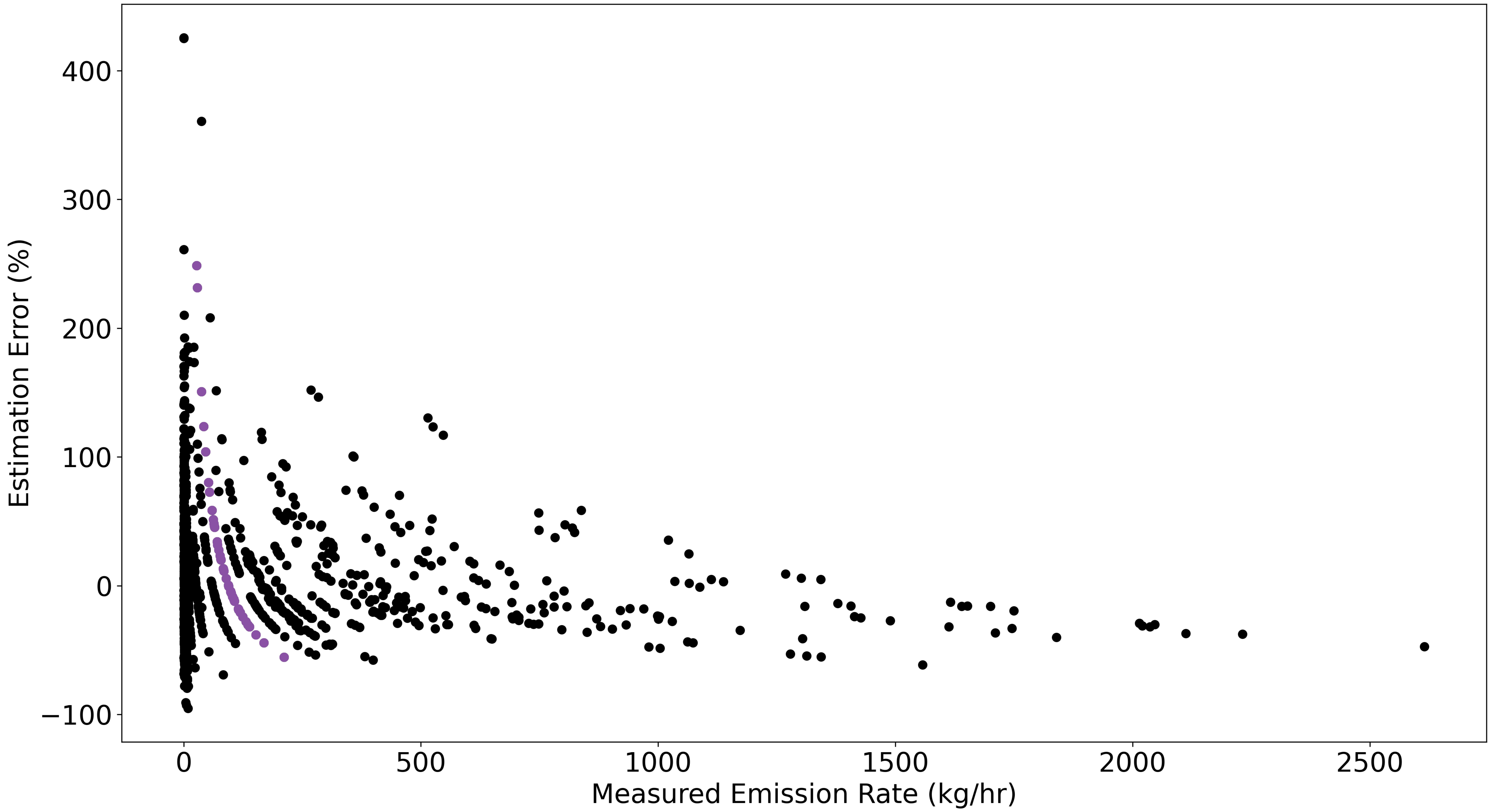
Background

$$\pi(Q|\tilde{Q}) = \int_{\hat{Q}} \pi_{\lambda_Q}\left(\frac{Q}{\hat{Q}}\right) \pi_{\kappa Q}\left(\frac{\hat{Q}}{f_B(\tilde{Q})}\right) \frac{1}{\hat{Q} f_B(\tilde{Q})} d\hat{Q}$$

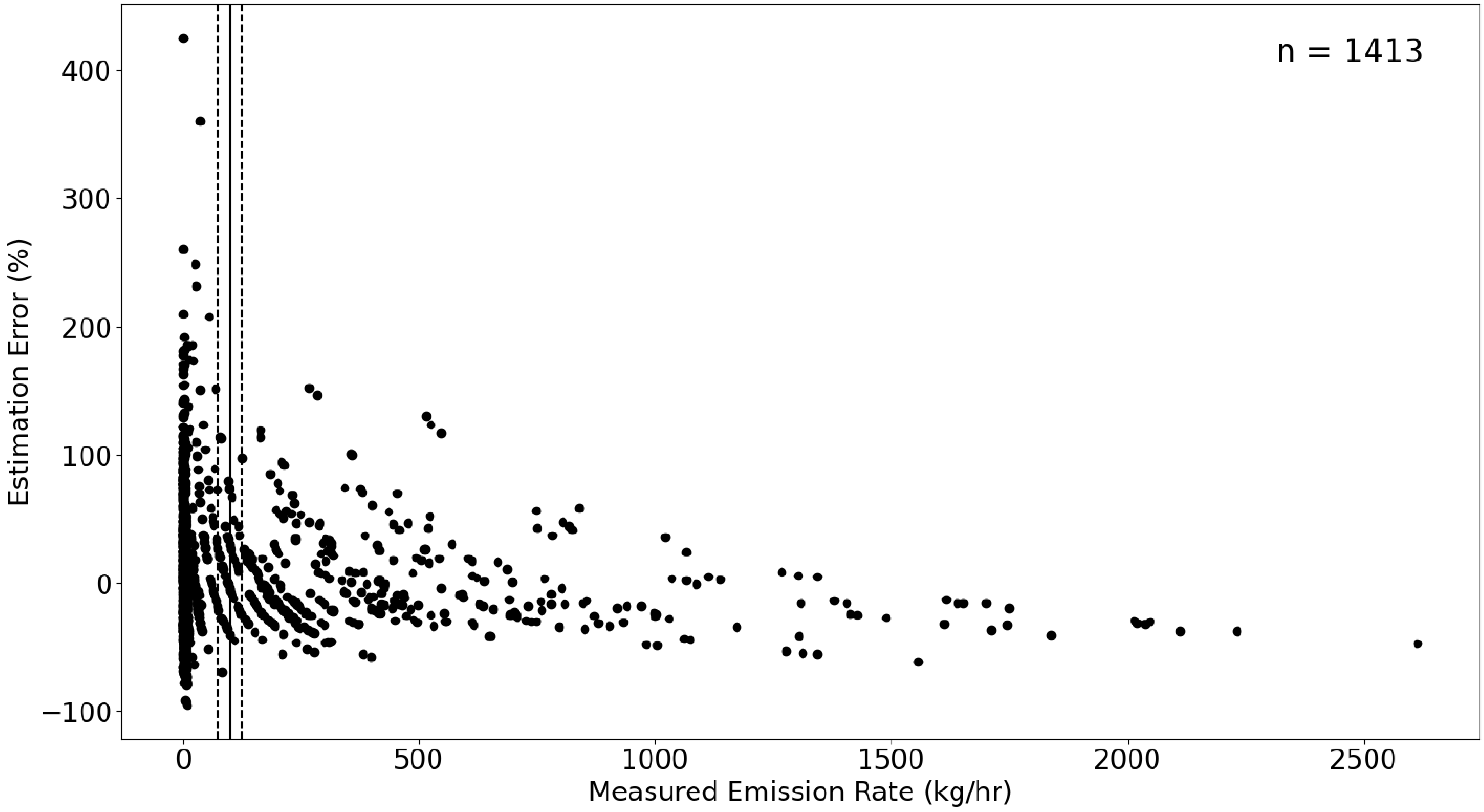
- Conrad, et al. investigated this in their 2023 paper
- Most specific model to date, relative error rate not dependent on measured emission rate
- Our goal: create a flexible framework that can use the best/most available data to estimate error distributions

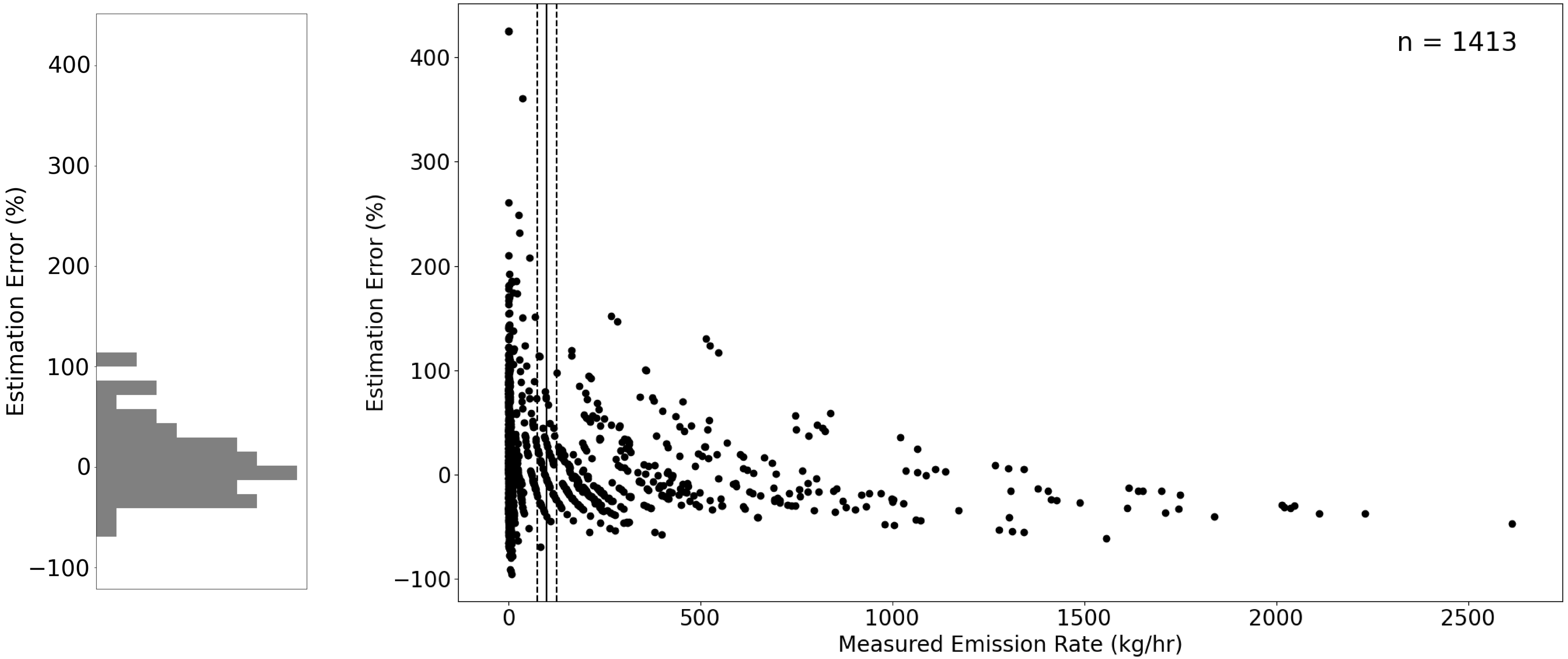
Source: Bradley M. Conrad, David R. Tyner, Matthew R. Johnson,
Robust probabilities of detection and quantification uncertainty for aerial methane detection: Examples for three airborne technologies,
Remote Sensing of Environment, Volume 288, 2023.


n = 1413
Estimation Error (%)
400
300
200
100
0
−100
0
500
1000
1500
2000
2500
Measured Emission Rate (kg/hr)

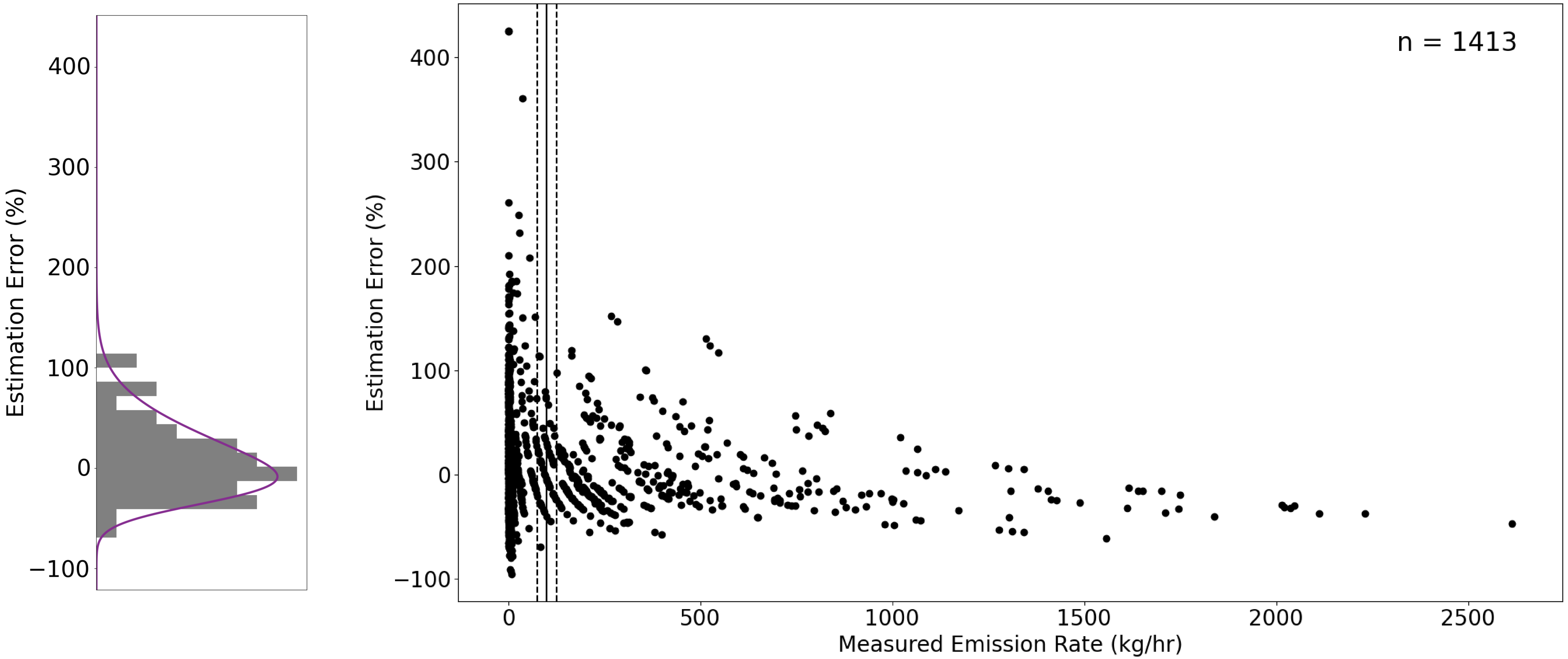


400
300
200
100
0
−100
Estimation Error (%)
0
500
1000
1500
2000
2500
Measured Emission Rate (kg/hr)


n = 1413
Estimation Error (%)
400
300
200
100
0
−100
0
500
1000
1500
2000
2500
Measured Emission Rate (kg/hr)

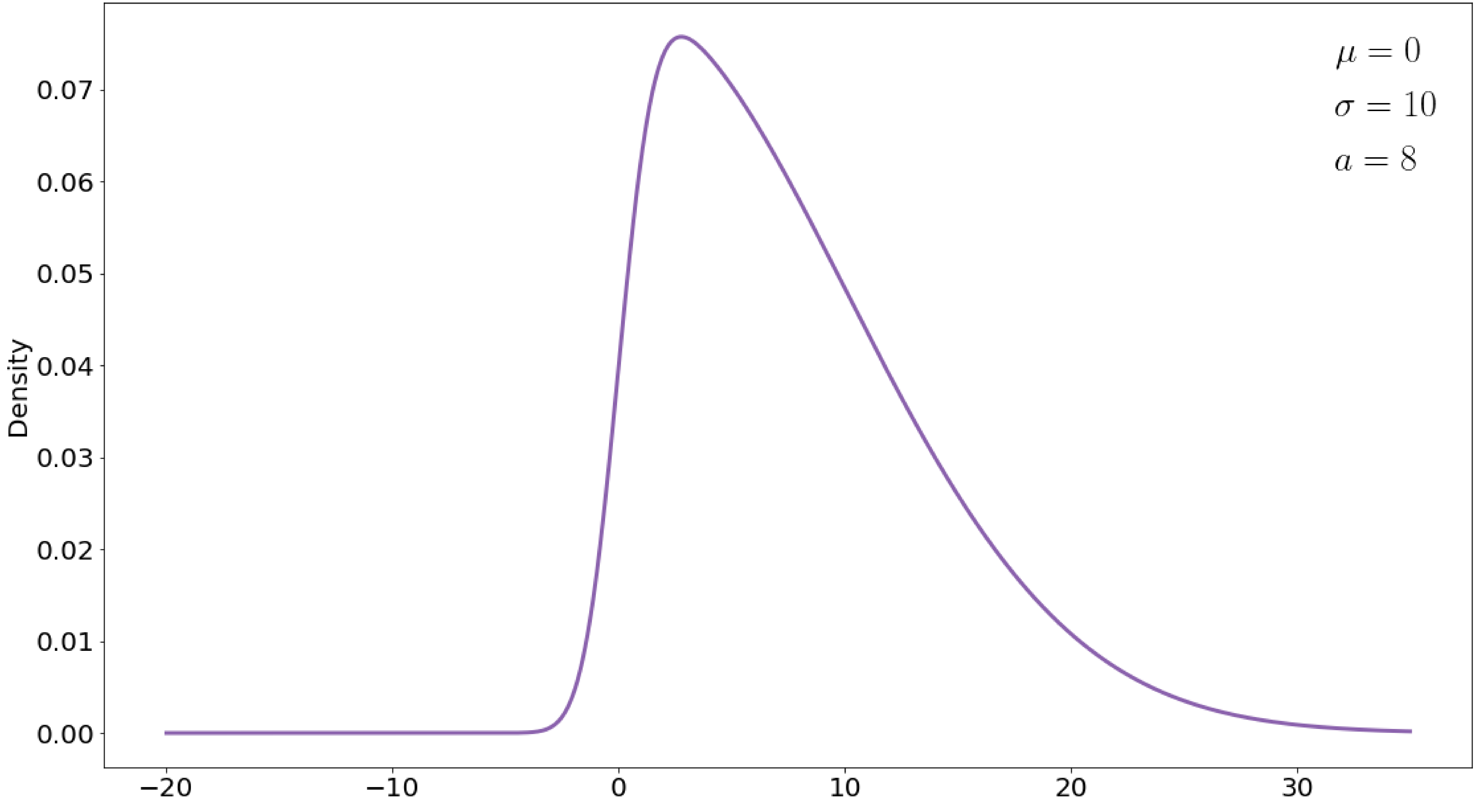


Estimation Error (%)
400
300
200
100
0
−100
n = 1413
Measured Emission Rate (kg/hr)
0
500
1000
1500
2000
2500


Estimation Error (%)
400
300
200
100
0
−100
n = 1413
Estimation Error (%)
400
300
200
100
0
−100
0
500
1000
1500
2000
2500
Measured Emission Rate (kg/hr)

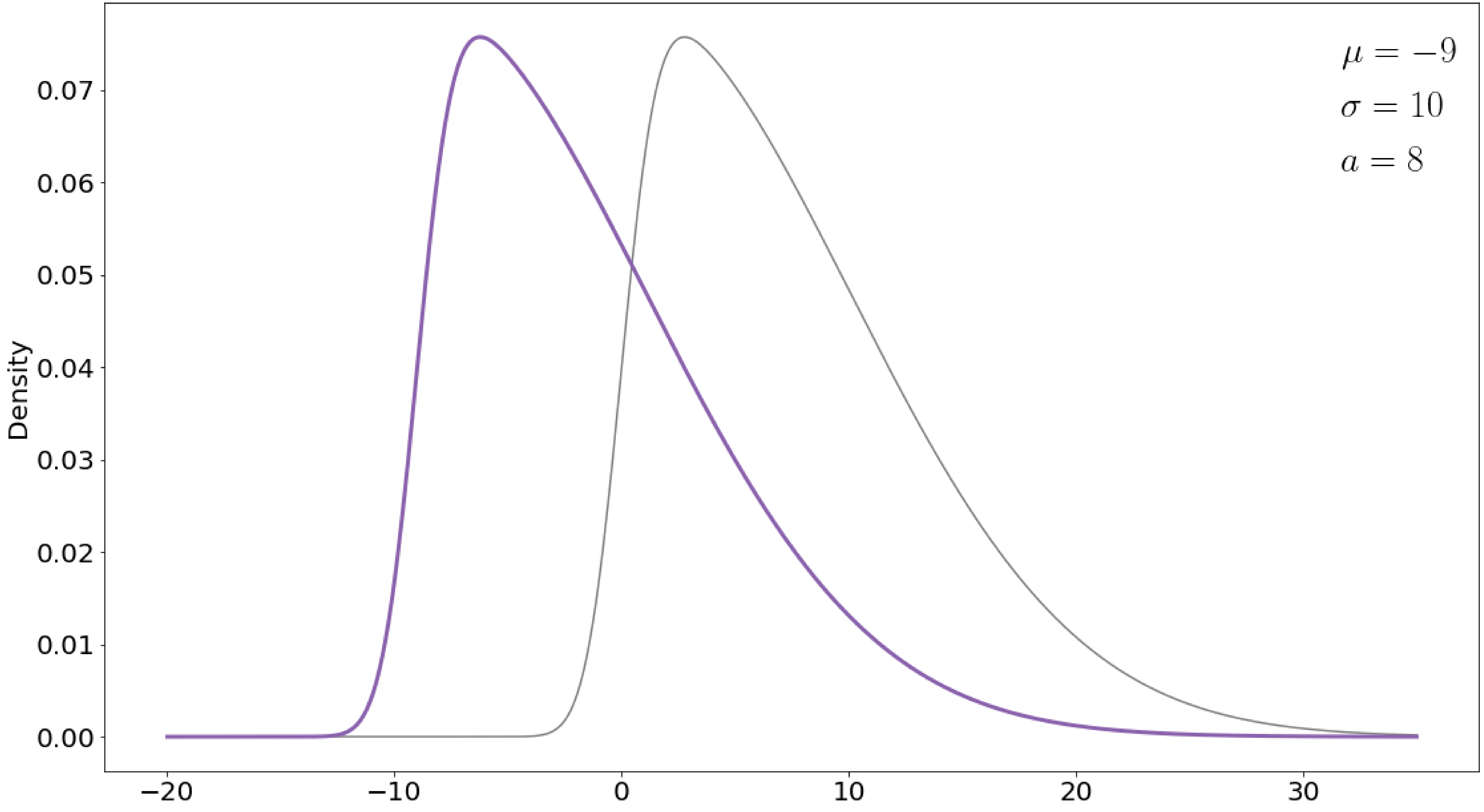
Location Parameter

$$f(x) = \frac{\varphi(\frac{x-\mu}{\sigma})\Phi(a\frac{x-\mu}{\sigma})}{\sigma\Phi(0)}$$



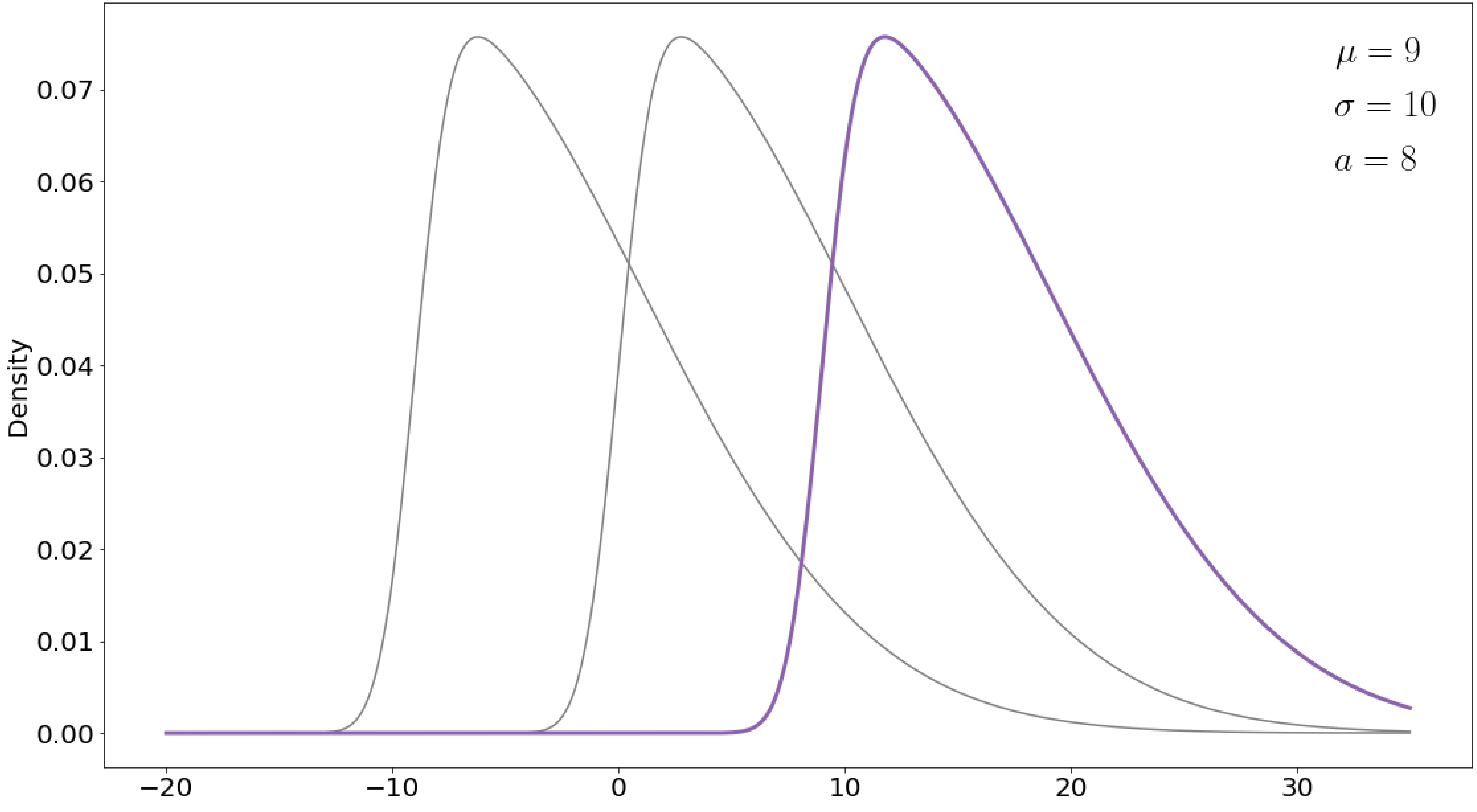
Location Parameter

$$f(x) = \frac{\varphi(\frac{x-\mu}{\sigma})\Phi(a\frac{x-\mu}{\sigma})}{\sigma\Phi(0)}$$



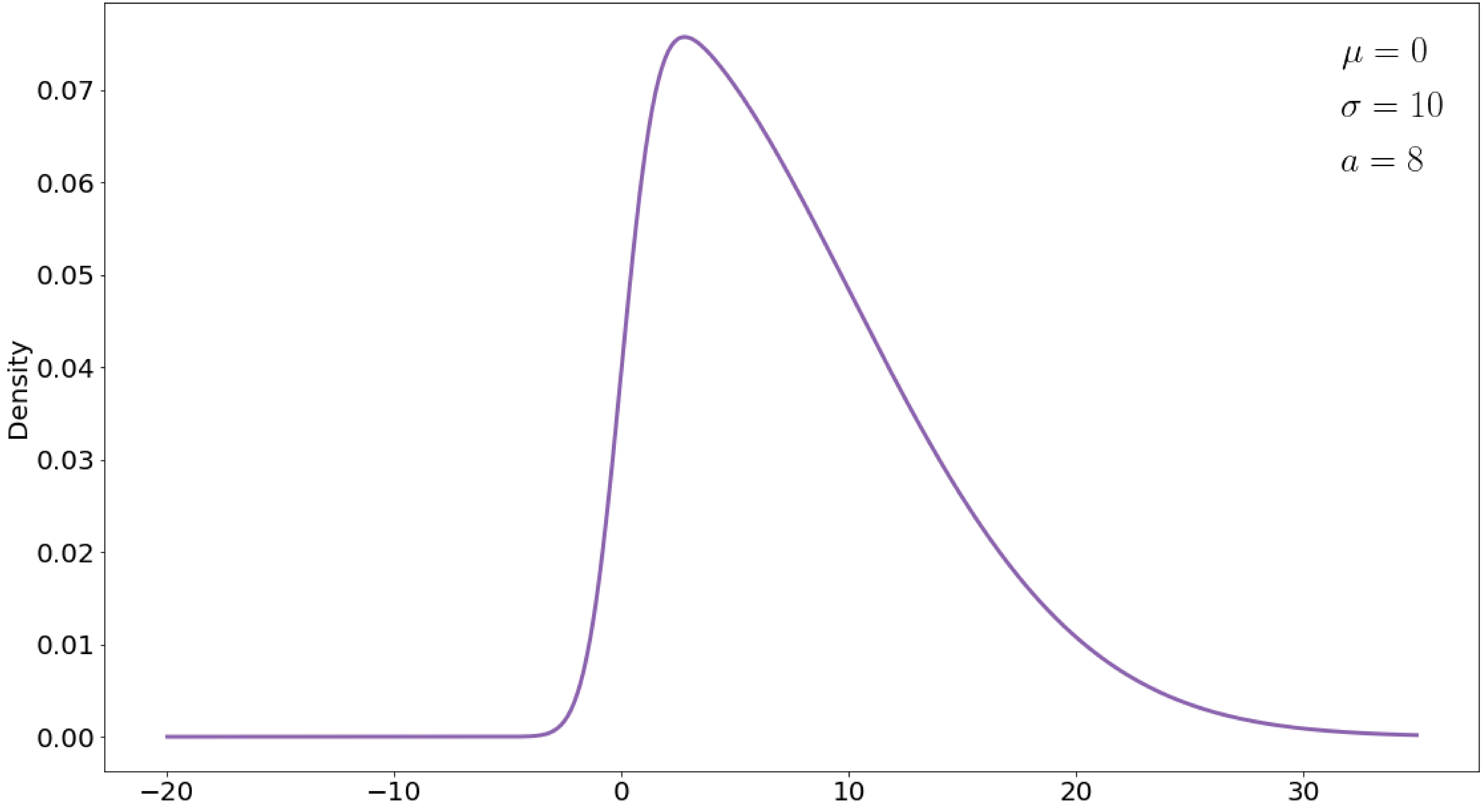
Location Parameter

$$f(x) = \frac{\varphi(\frac{x-\mu}{\sigma})\Phi(a\frac{x-\mu}{\sigma})}{\sigma\Phi(0)}$$



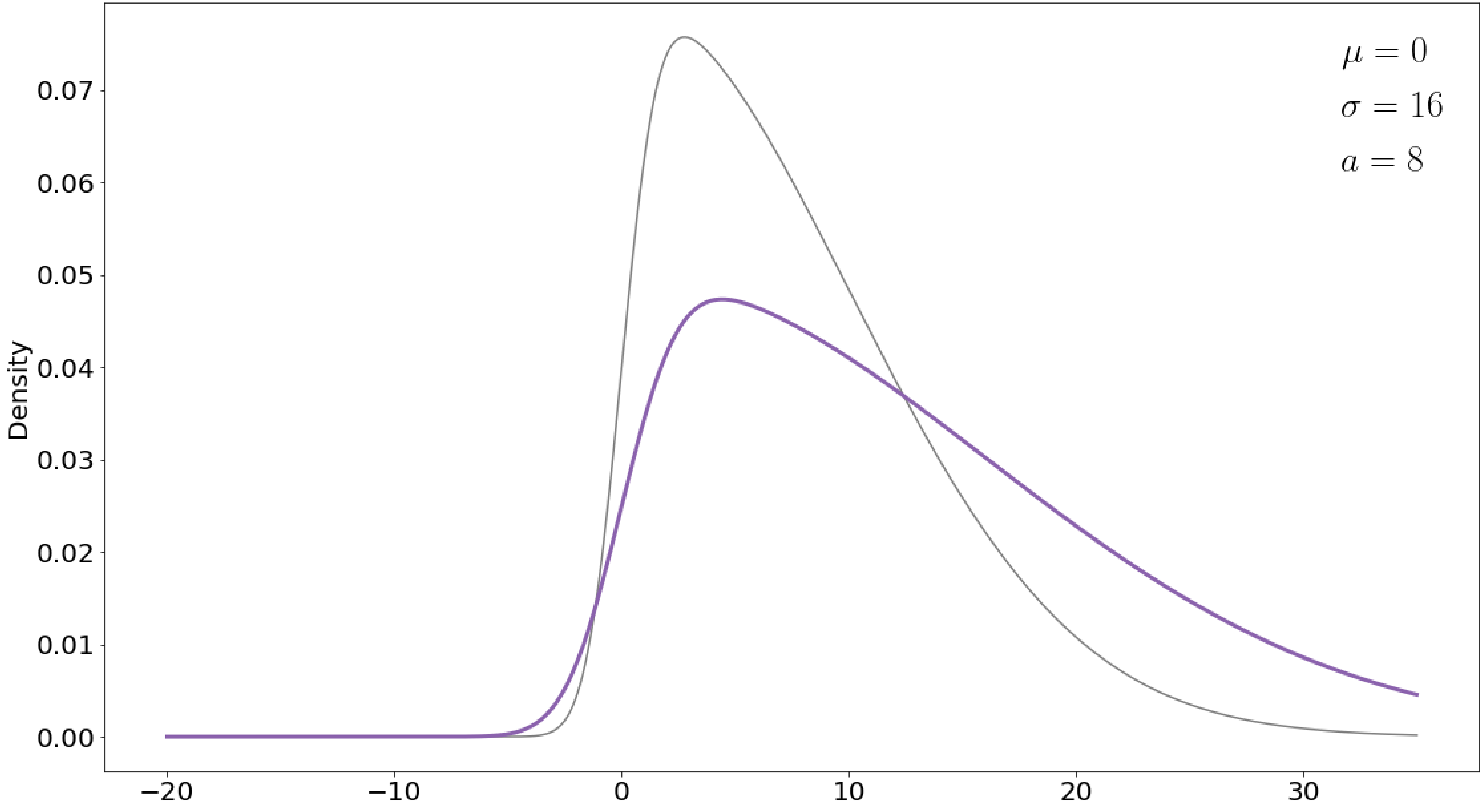
Scale Parameter

$$f(x) = \frac{\varphi(\frac{x-\mu}{\sigma})\Phi(a\frac{x-\mu}{\sigma})}{\sigma\Phi(0)}$$



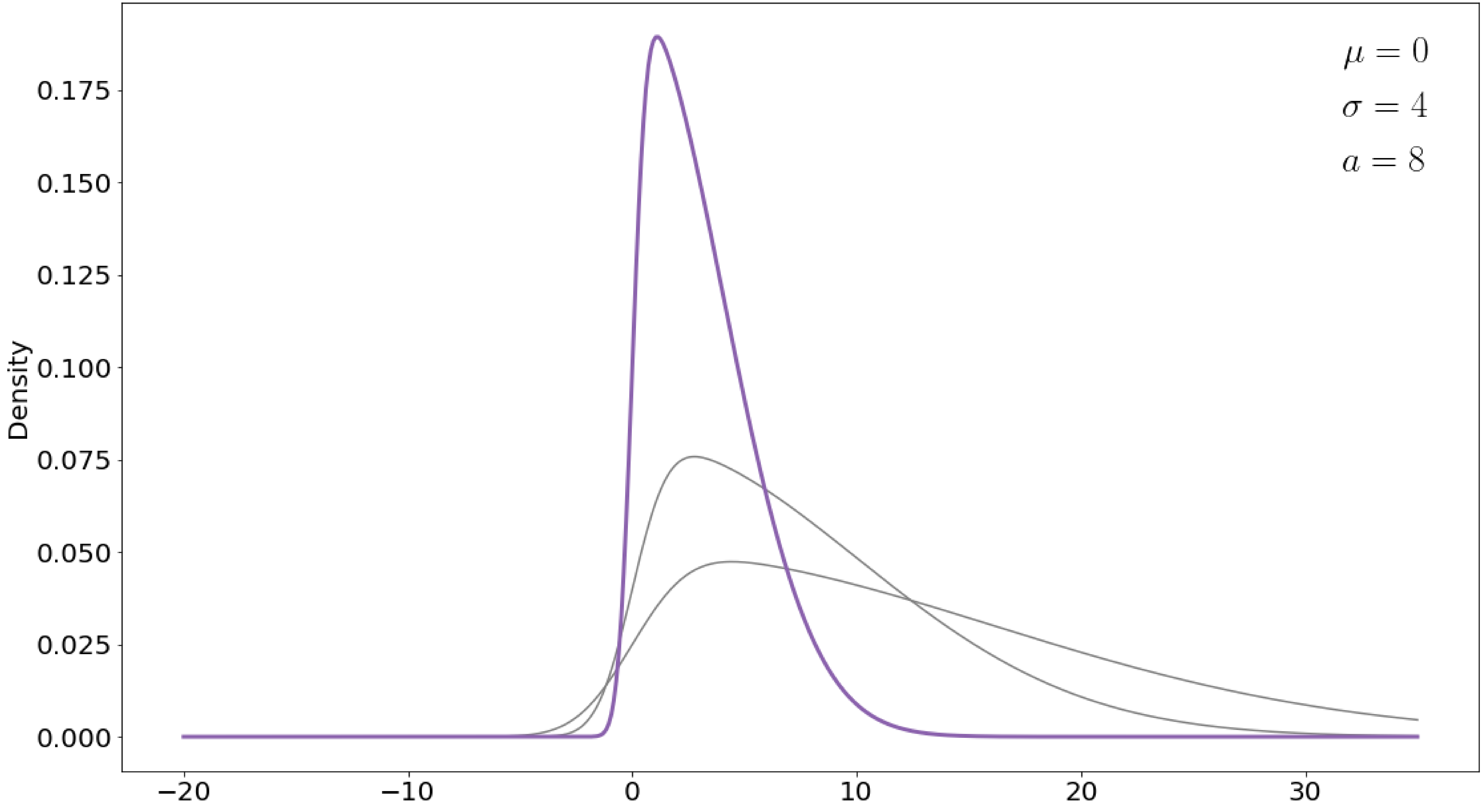
Scale Parameter

$$f(x) = \frac{\varphi(\frac{x-\mu}{\sigma})\Phi(a\frac{x-\mu}{\sigma})}{\sigma\Phi(0)}$$



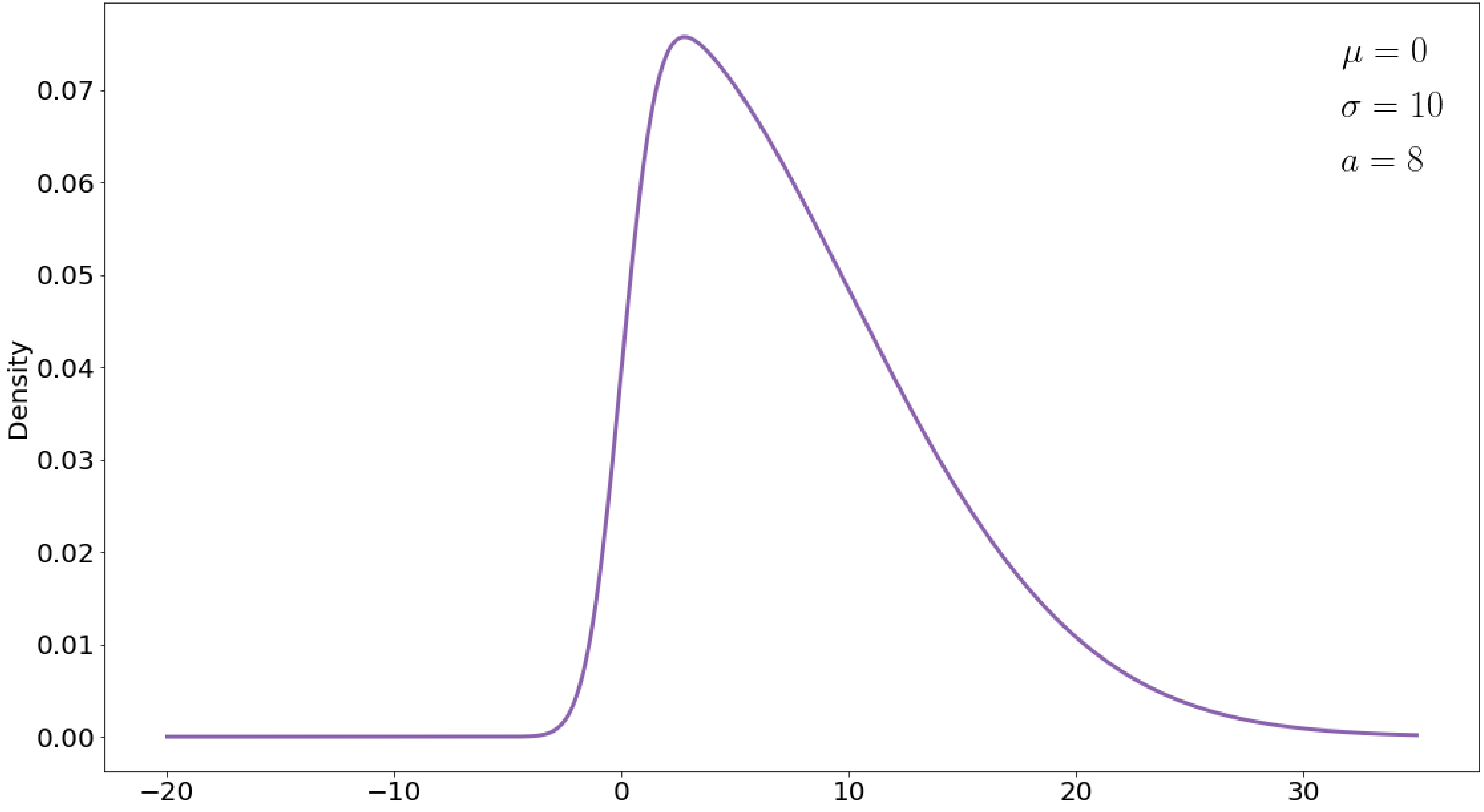
Scale Parameter

$$f(x) = \frac{\varphi(\frac{x-\mu}{\sigma})\Phi(a\frac{x-\mu}{\sigma})}{\sigma\Phi(0)}$$



Skewness Parameter

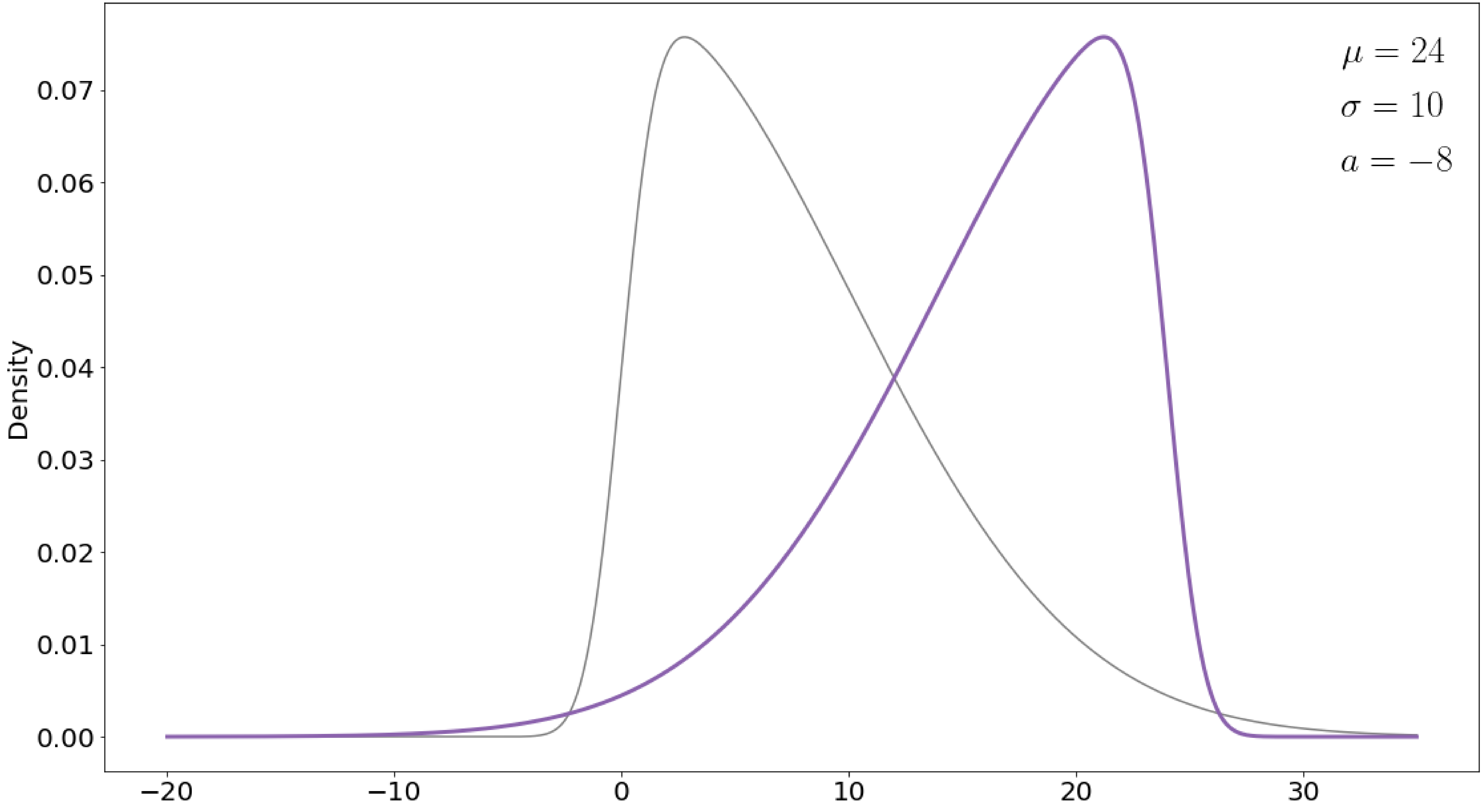
$$f(x) = \frac{\varphi(\frac{x-\mu}{\sigma})\Phi(a\frac{x-\mu}{\sigma})}{\sigma\Phi(0)}$$



COLORADOSCHOOLOFMINES
EARTH • ENERGY • ENVIRONMENT

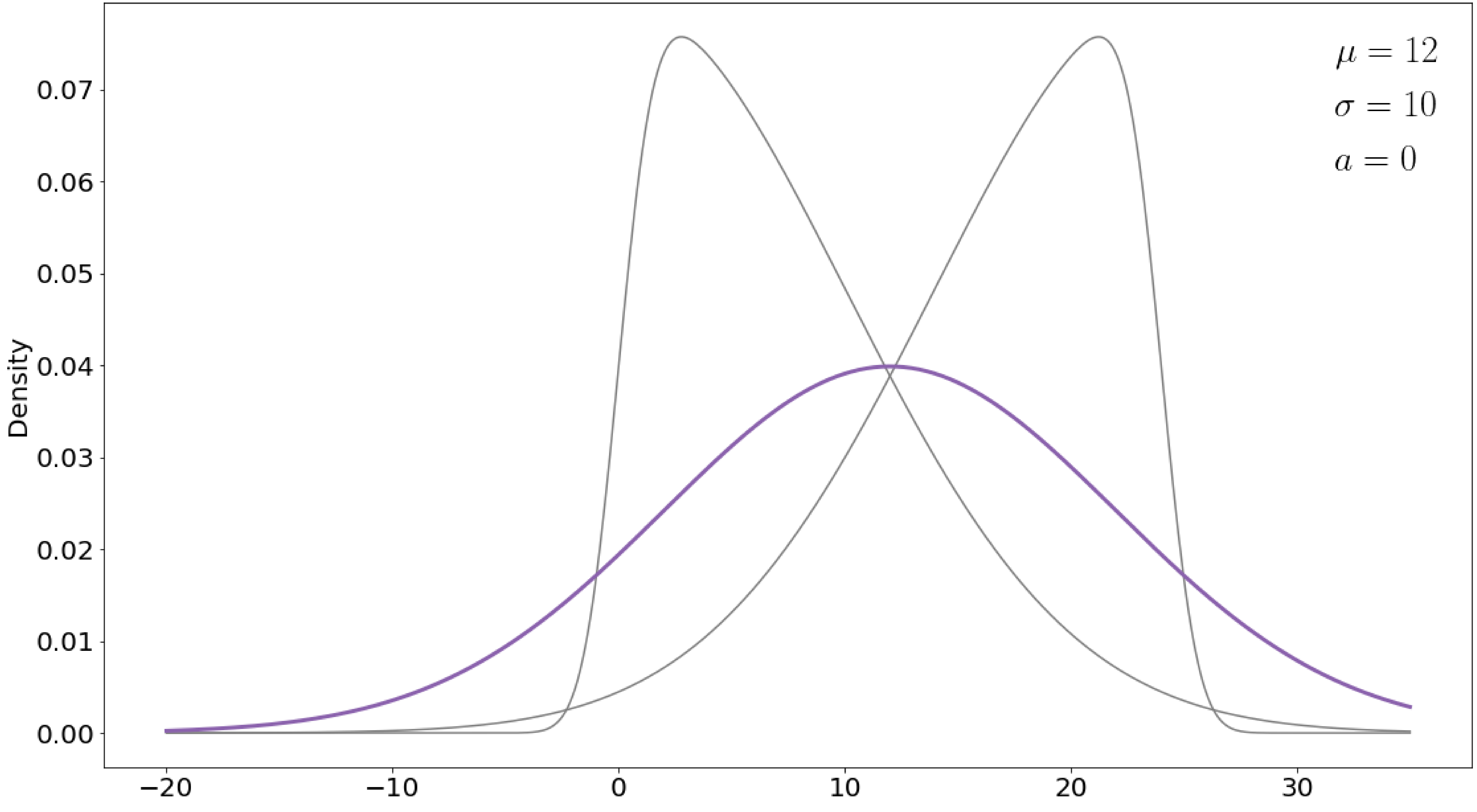
Scale Parameter

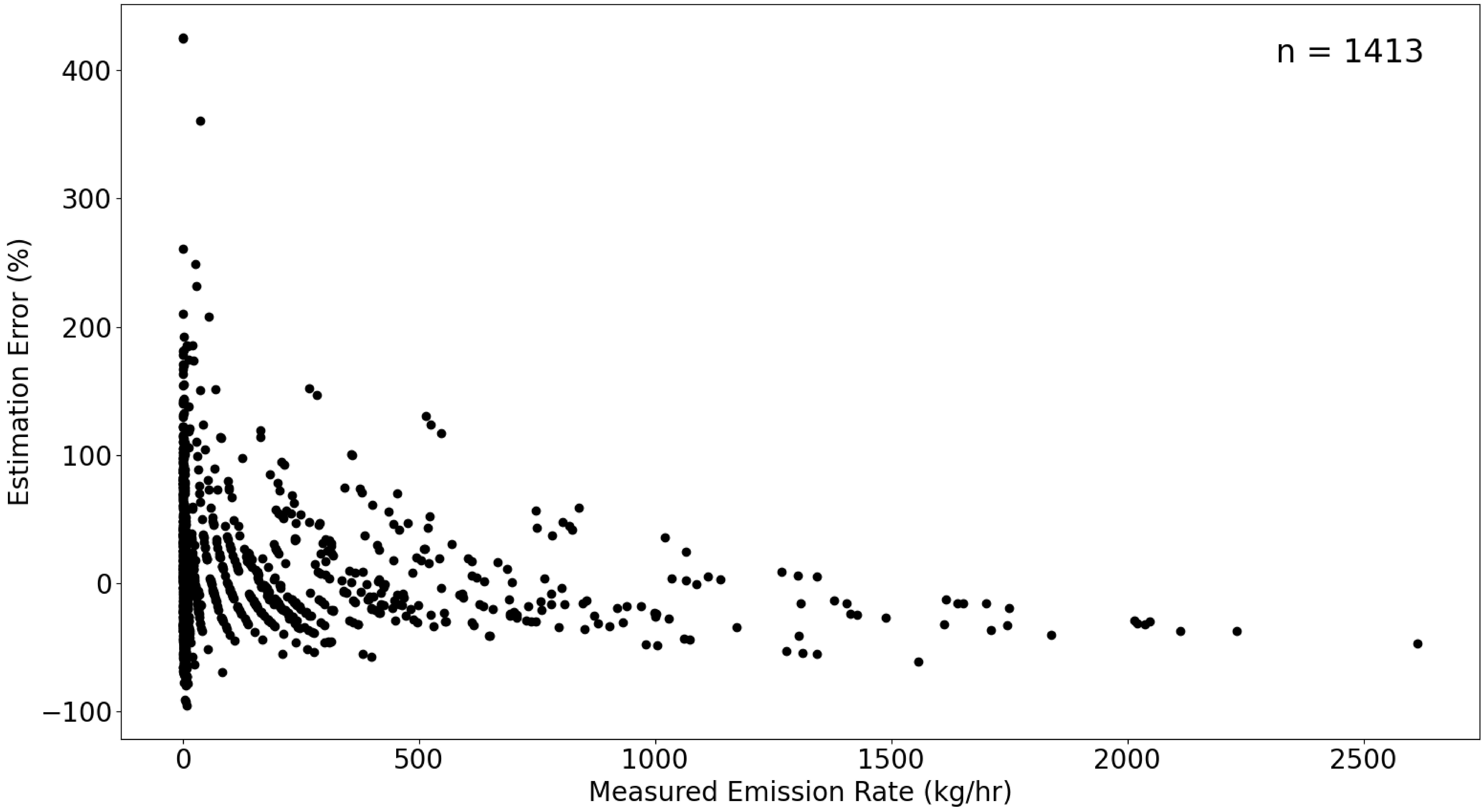
$$f(x) = \frac{\varphi(\frac{x-\mu}{\sigma})\Phi(a\frac{x-\mu}{\sigma})}{\sigma\Phi(0)}$$



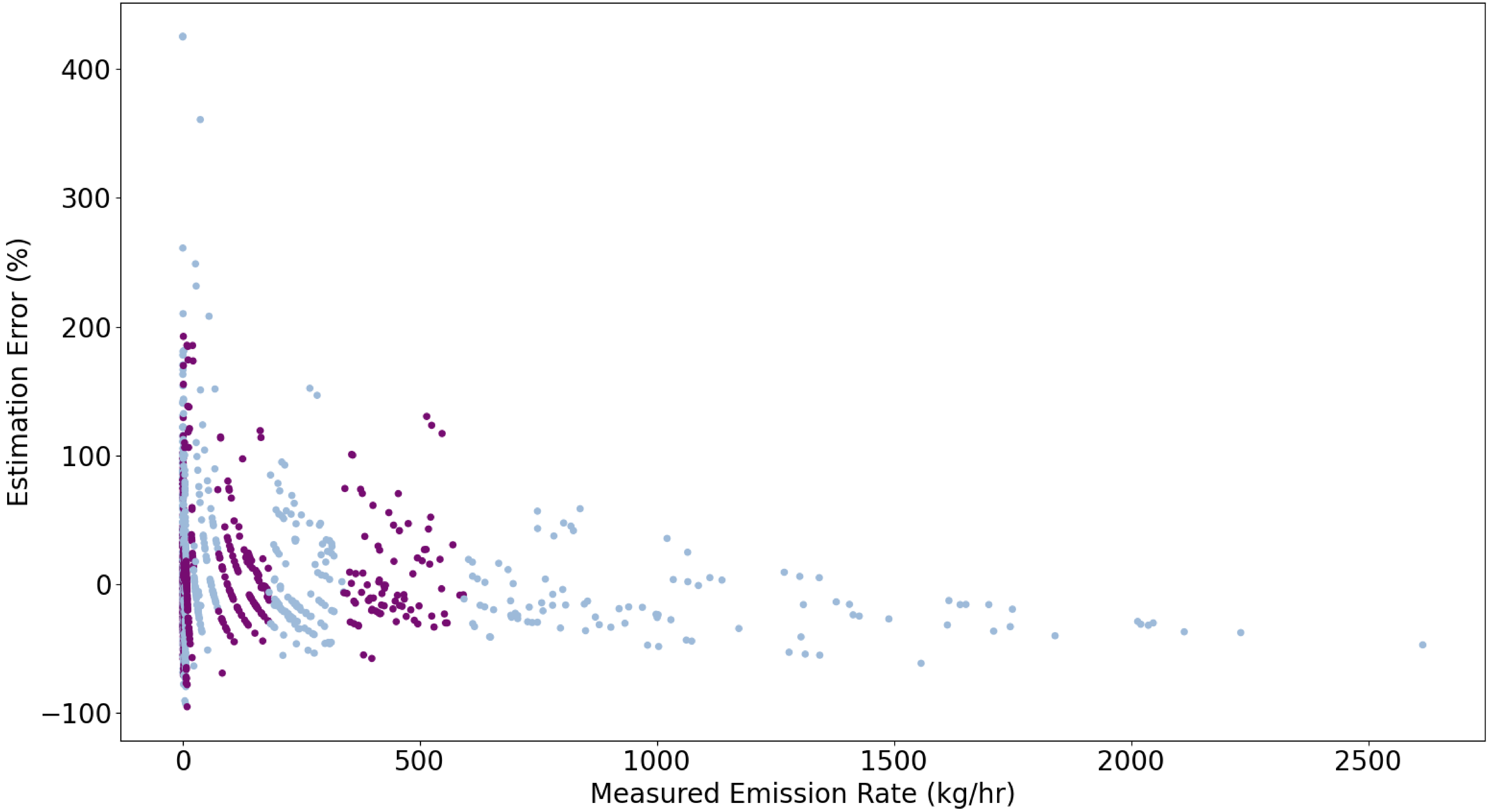
Scale Parameter

$$f(x) = \frac{\varphi(\frac{x-\mu}{\sigma})\Phi(a\frac{x-\mu}{\sigma})}{\sigma\Phi(0)}$$

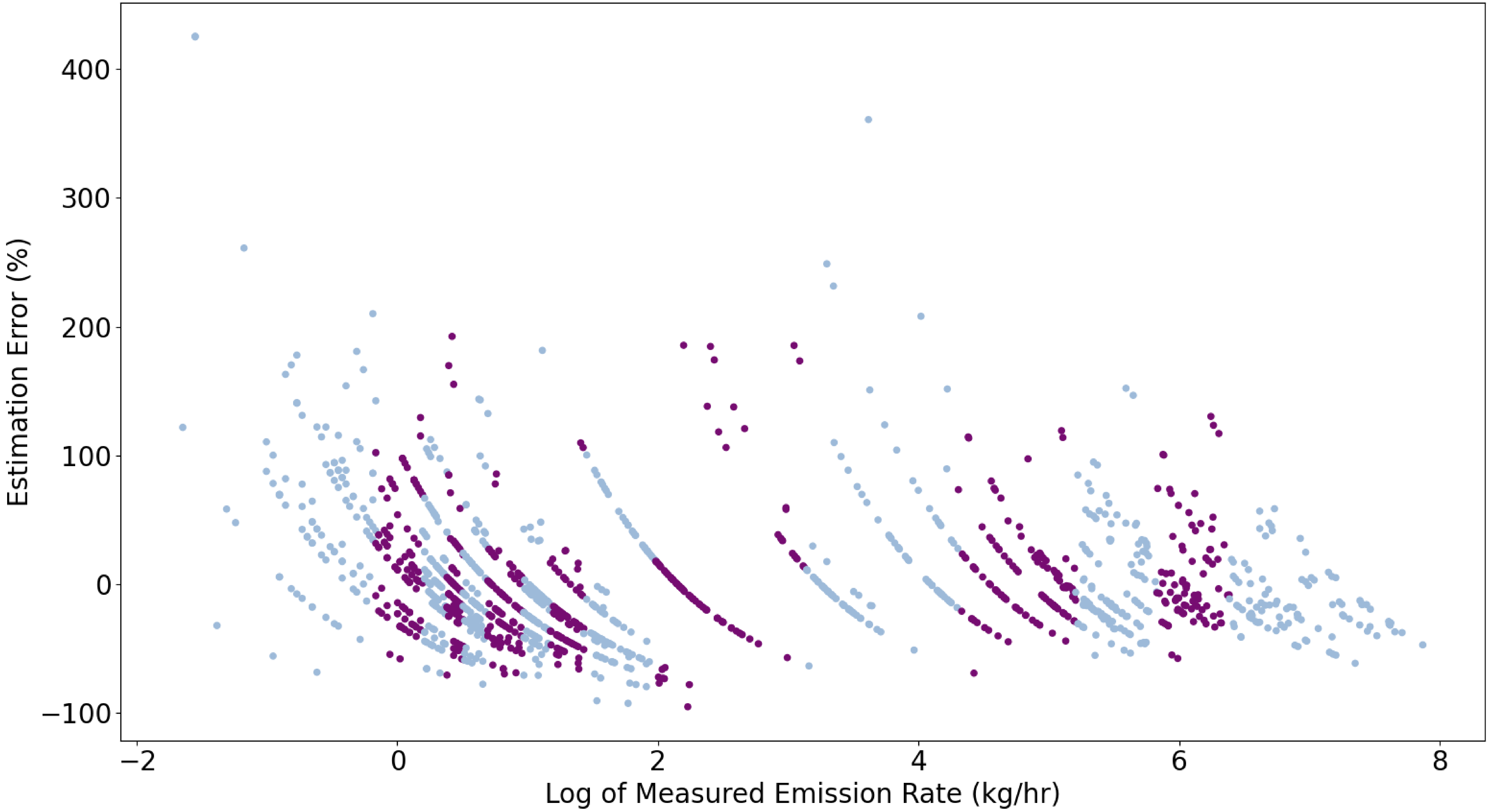



n = 1413
Estimation Error (%)
400
300
200
100
0
−100
0
500
1000
1500
2000
2500
Measured Emission Rate (kg/hr)

Method 1: Binning

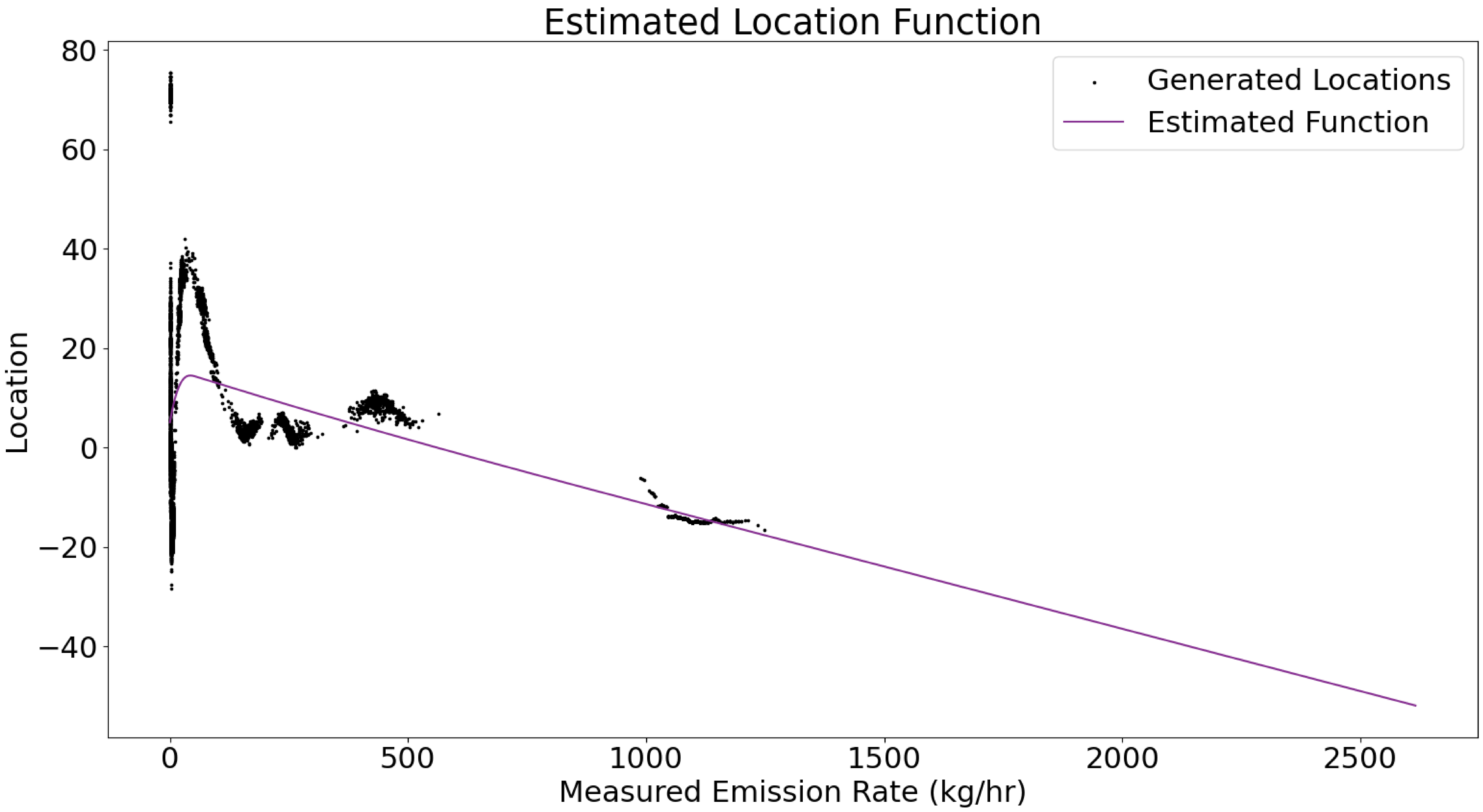


Method 1: Binning



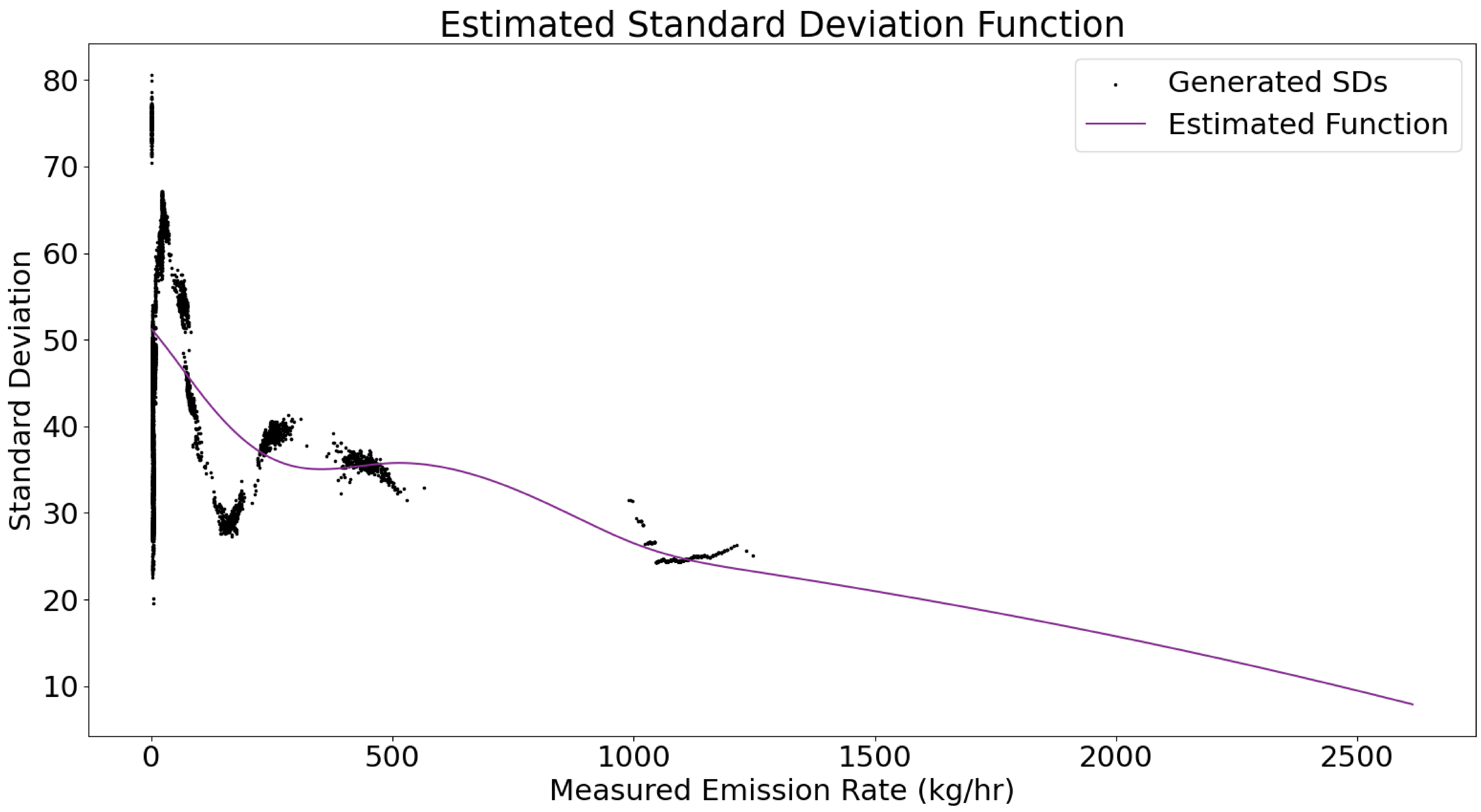
Method 1: Binning

$$f(x) = \frac{\varphi(\frac{x-\mu}{\sigma})\Phi(a\frac{x-\mu}{\sigma})}{\sigma\Phi(0)}$$



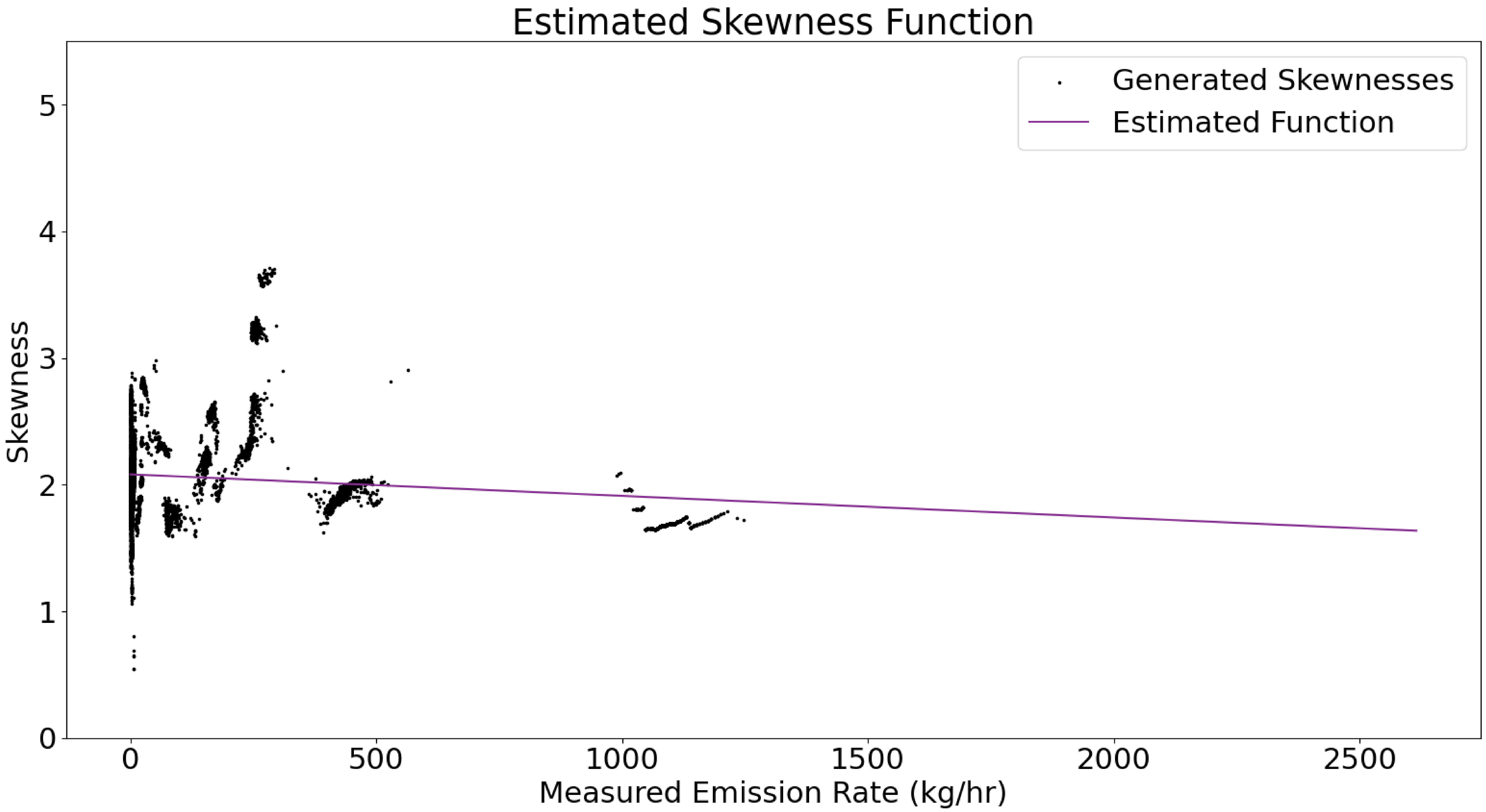
Method 1: Binning

$$f(x) = \frac{\varphi(\frac{x-\mu}{\sigma})\Phi(a\frac{x-\mu}{\sigma})}{\sigma\Phi(0)}$$



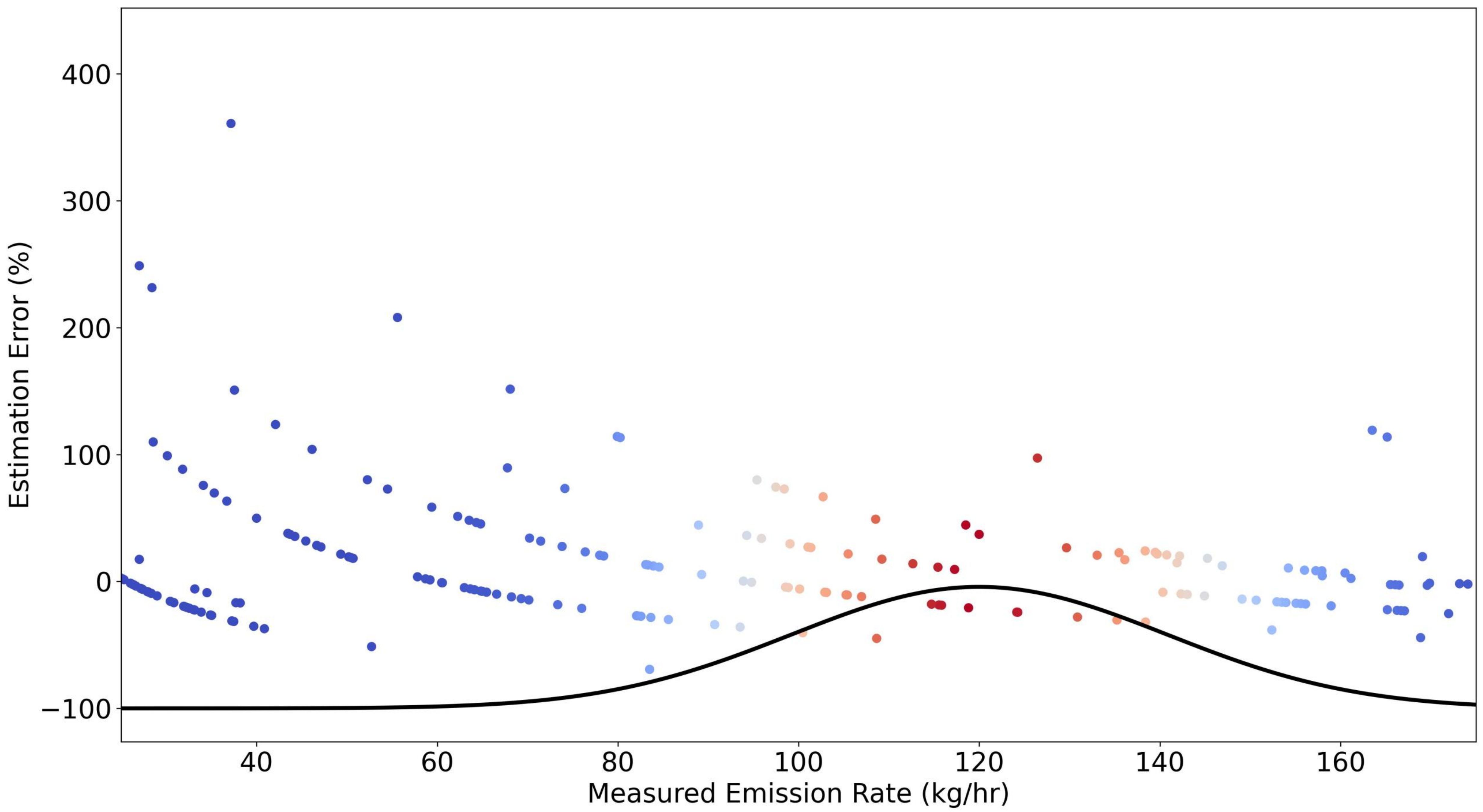
Method 1: Binning

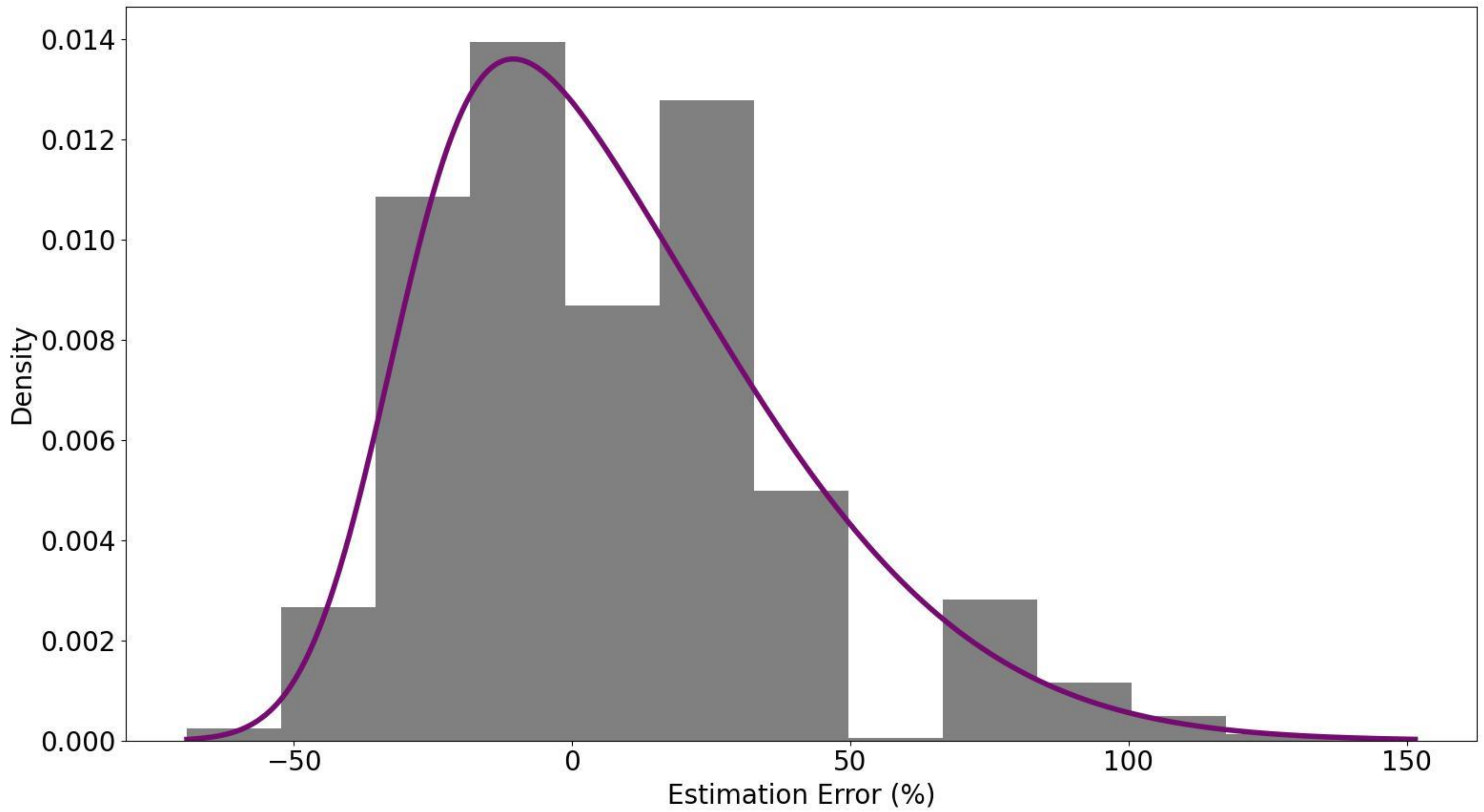
$$f(x) = \frac{\varphi(\frac{x-\mu}{\sigma})\Phi(a\frac{x-\mu}{\sigma})}{\sigma\Phi(0)}$$



Method 2: Kernel-Based

Method 2: Kernel-Based

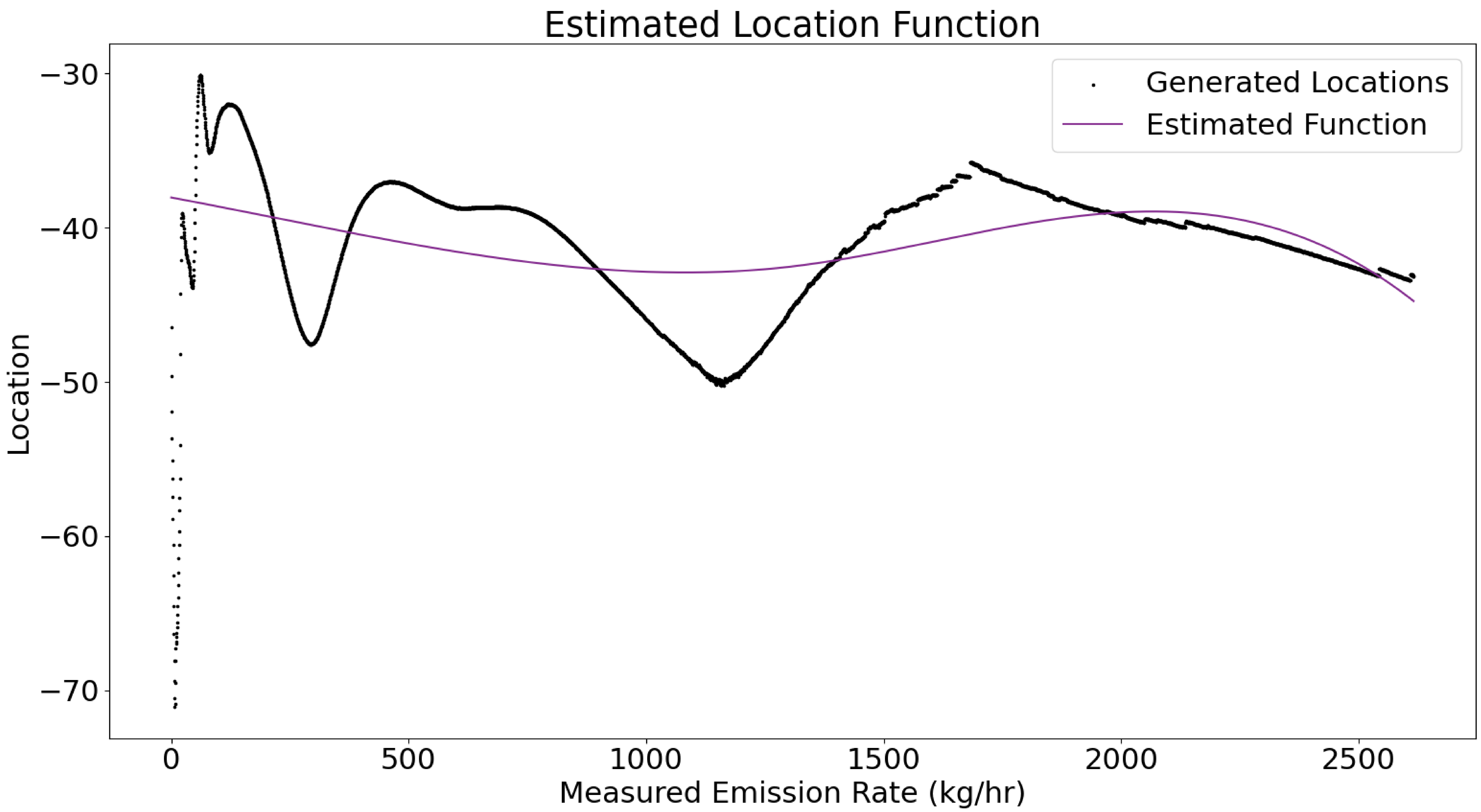




COLORADOSCHOOLOFMINES
EARTH ENERGY ENVIRONMENT

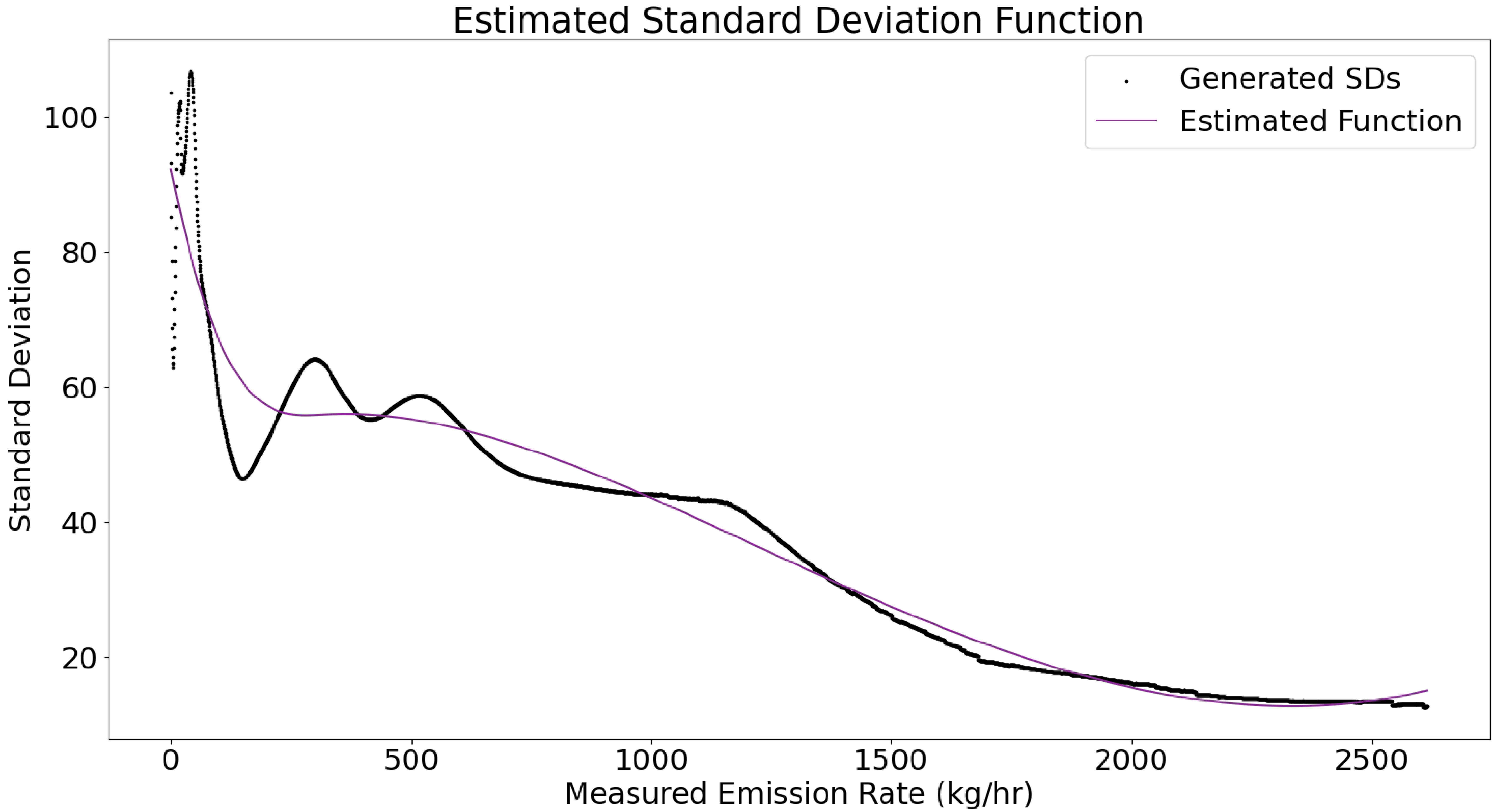
Method 2: Kernel-Based

$$f(x) = \frac{\varphi(\frac{x-\mu}{\sigma})\Phi(a\frac{x-\mu}{\sigma})}{\sigma\Phi(0)}$$



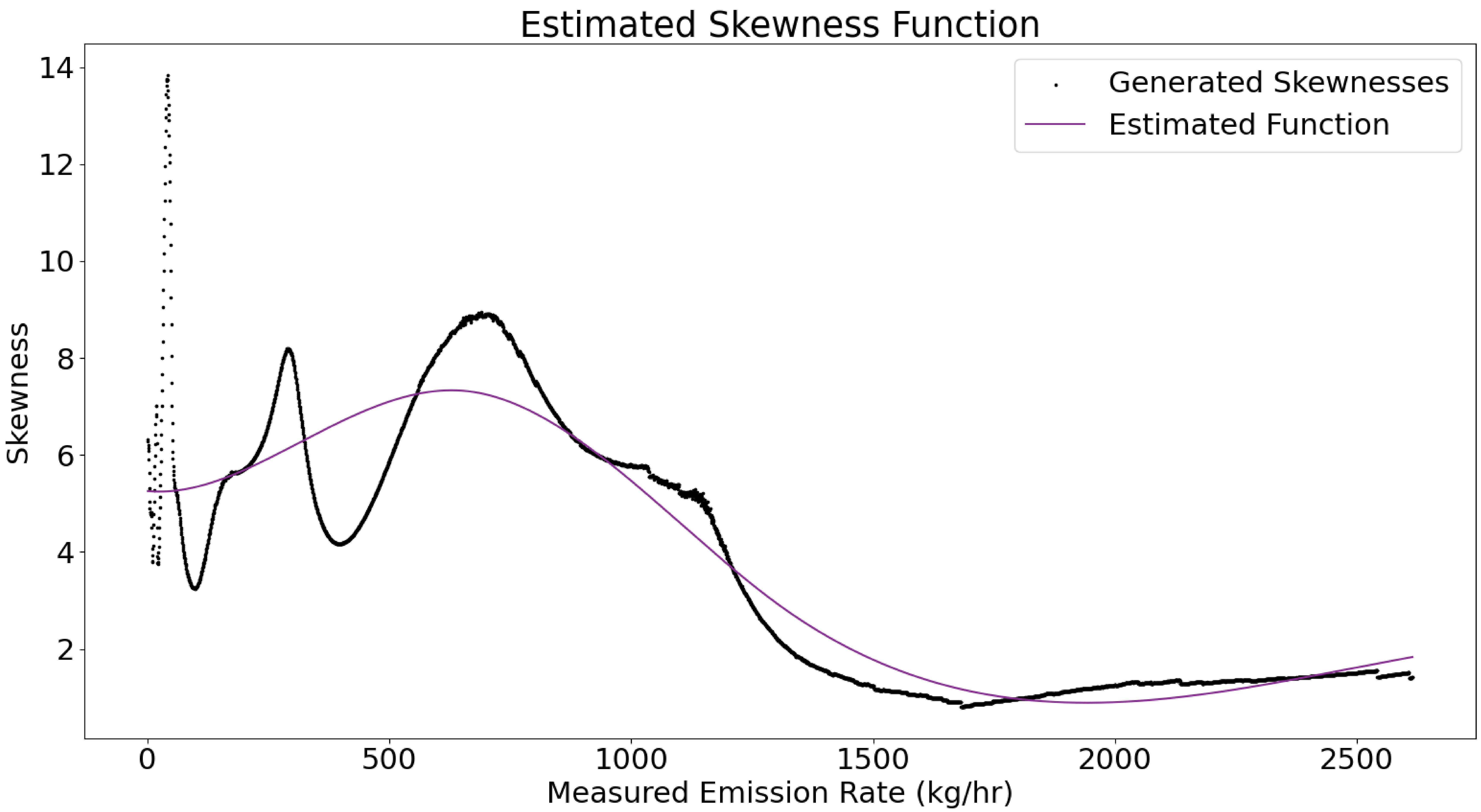
Method 2: Kernel-Based

$$f(x) = \frac{\varphi(\frac{x-\mu}{\sigma})\Phi(a\frac{x-\mu}{\sigma})}{\sigma\Phi(0)}$$



Method 2: Kernel-Based

$$f(x) = \frac{\varphi(\frac{x-\mu}{\sigma})\Phi(a\frac{x-\mu}{\sigma})}{\sigma\Phi(0)}$$

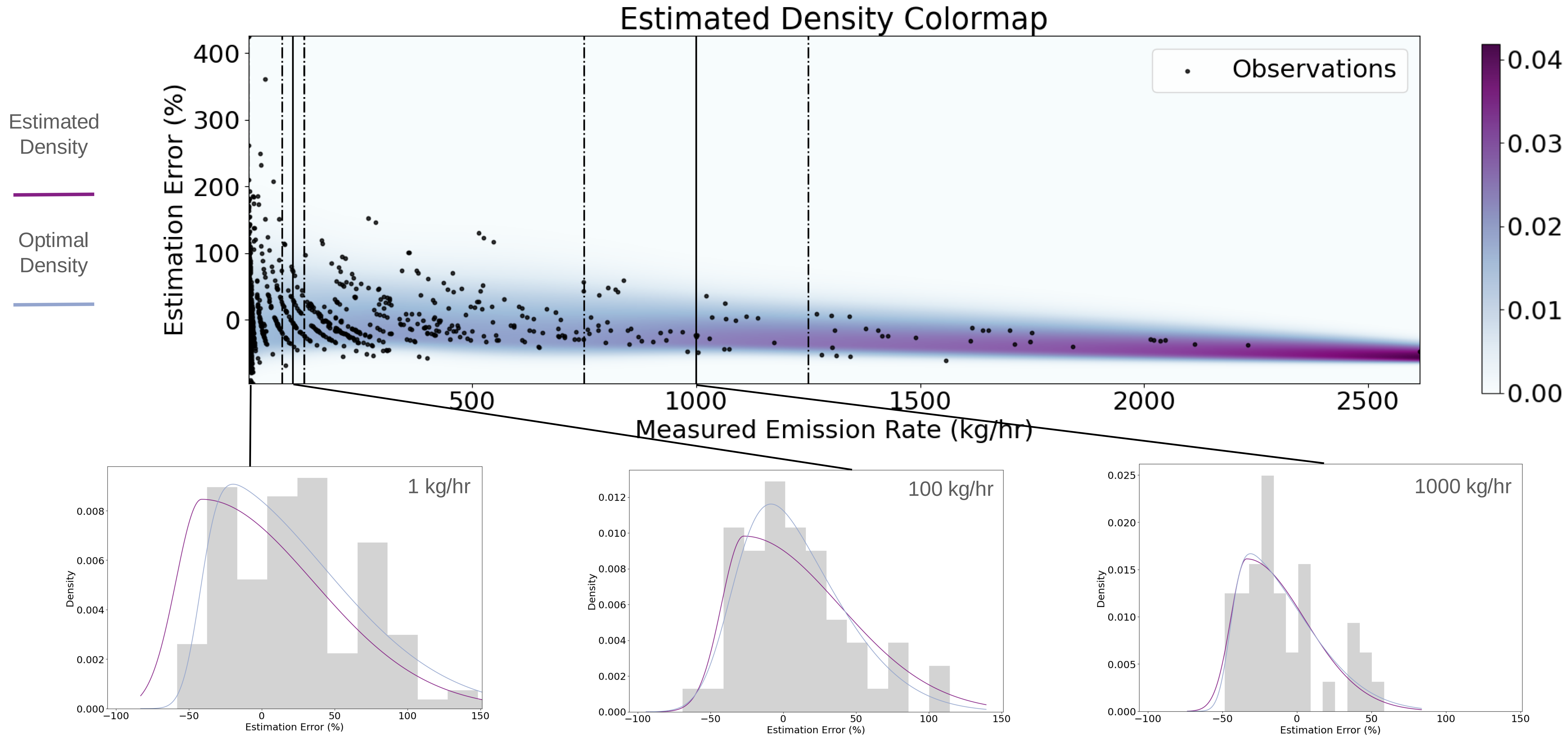


Framework Output

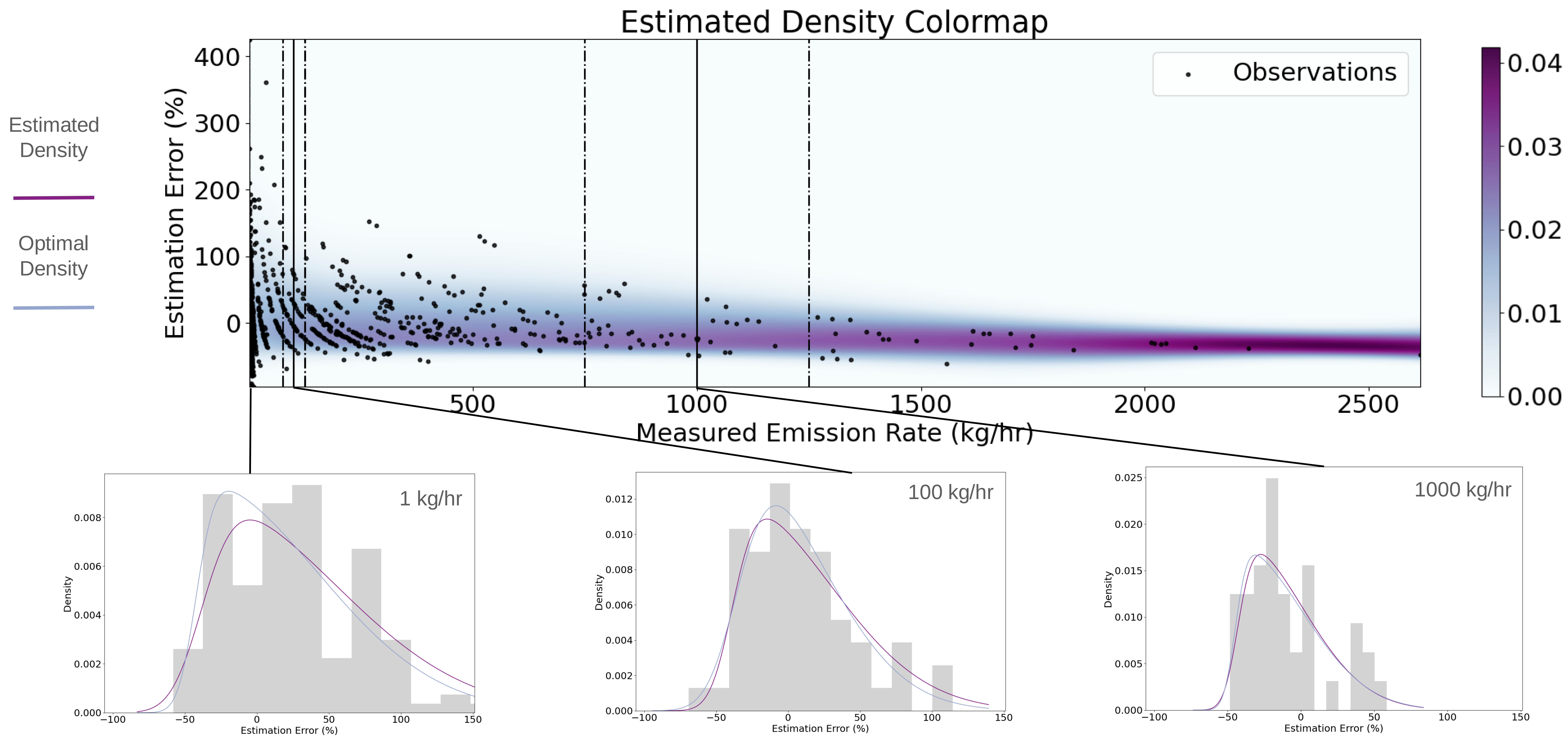
$$f(x) = \frac{\varphi(\frac{x-\mu}{\sigma})\Phi(a\frac{x-\mu}{\sigma})}{\sigma\Phi(0)}$$

- Both methods generate 3 functions (of measured emission rate):
  - Location
  - Standard Deviation
  - Skewness
- From these, a skew normal distribution can be generated for any value of measured emission rate

Method 1: Binning

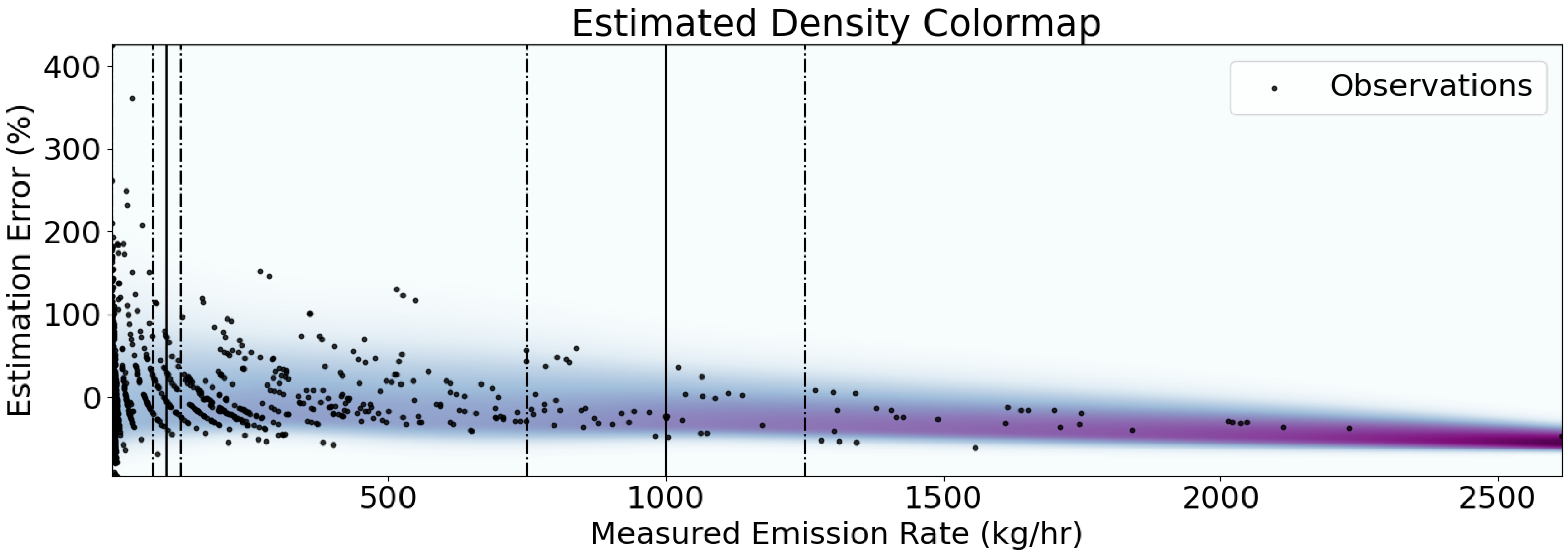


Method 2: Kernel-Based

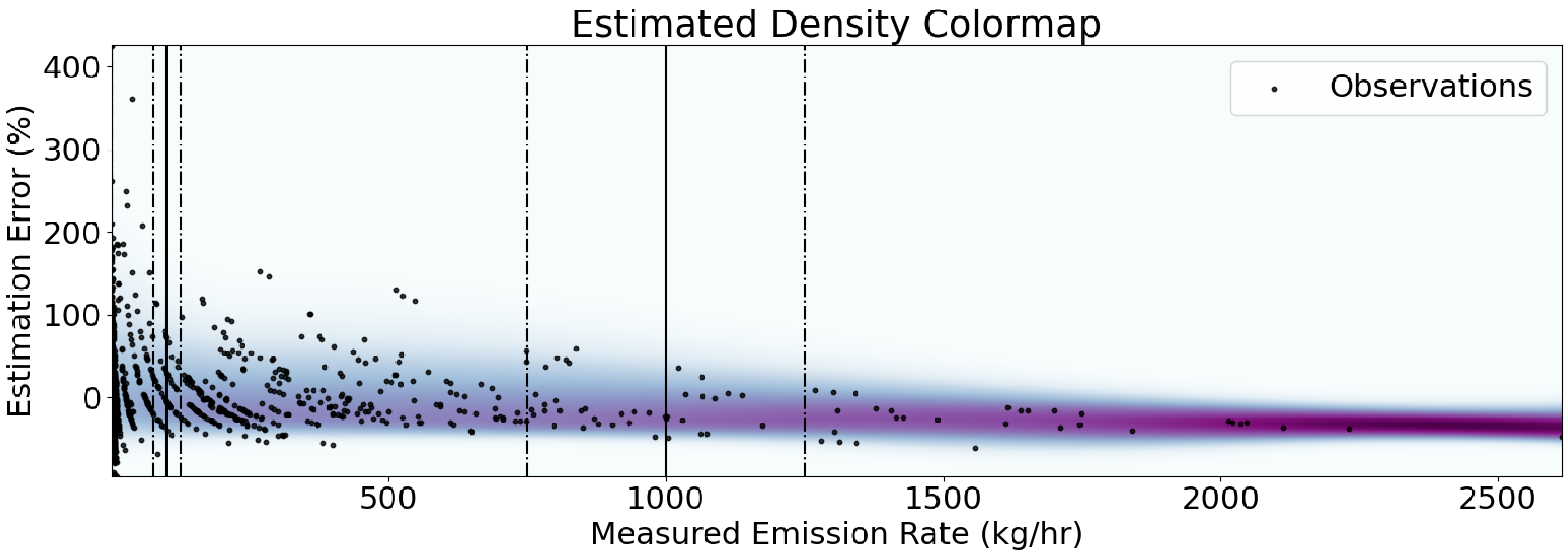


Comparison

Method 1: Binning

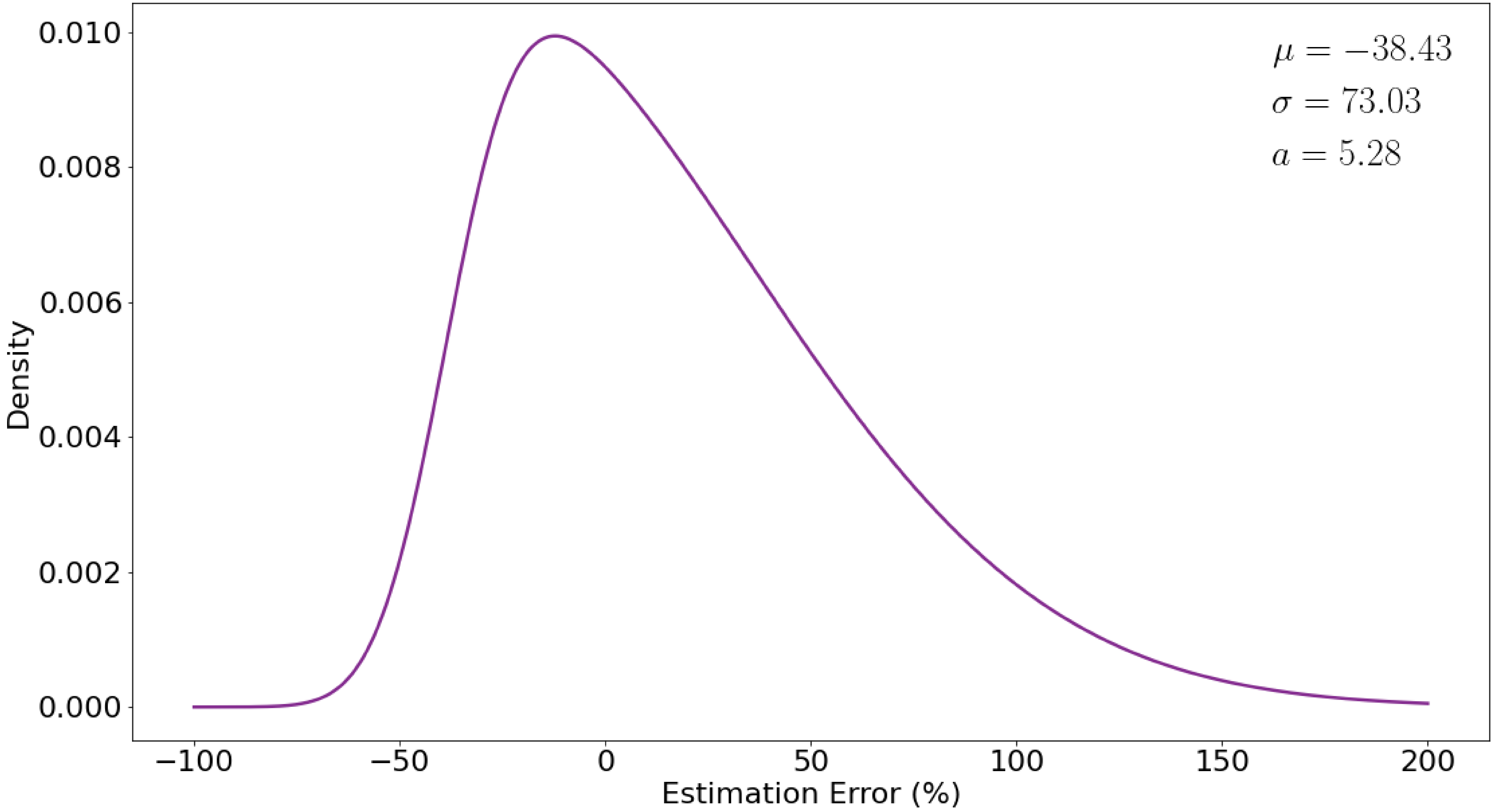


Method 2: Kernel-Based



Use Example

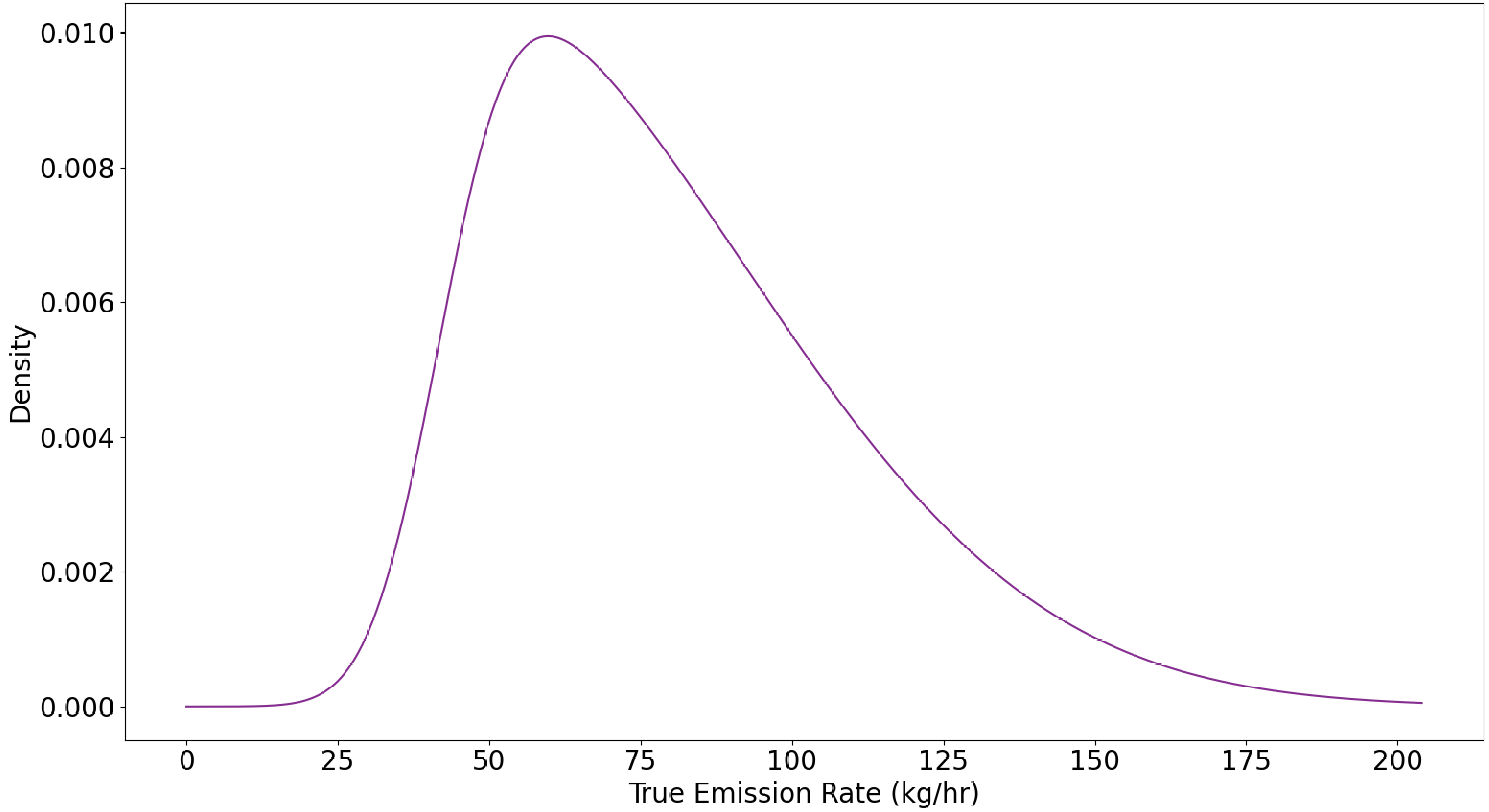
Measured Emission Rate = 68 kg/hr



Real-World Example

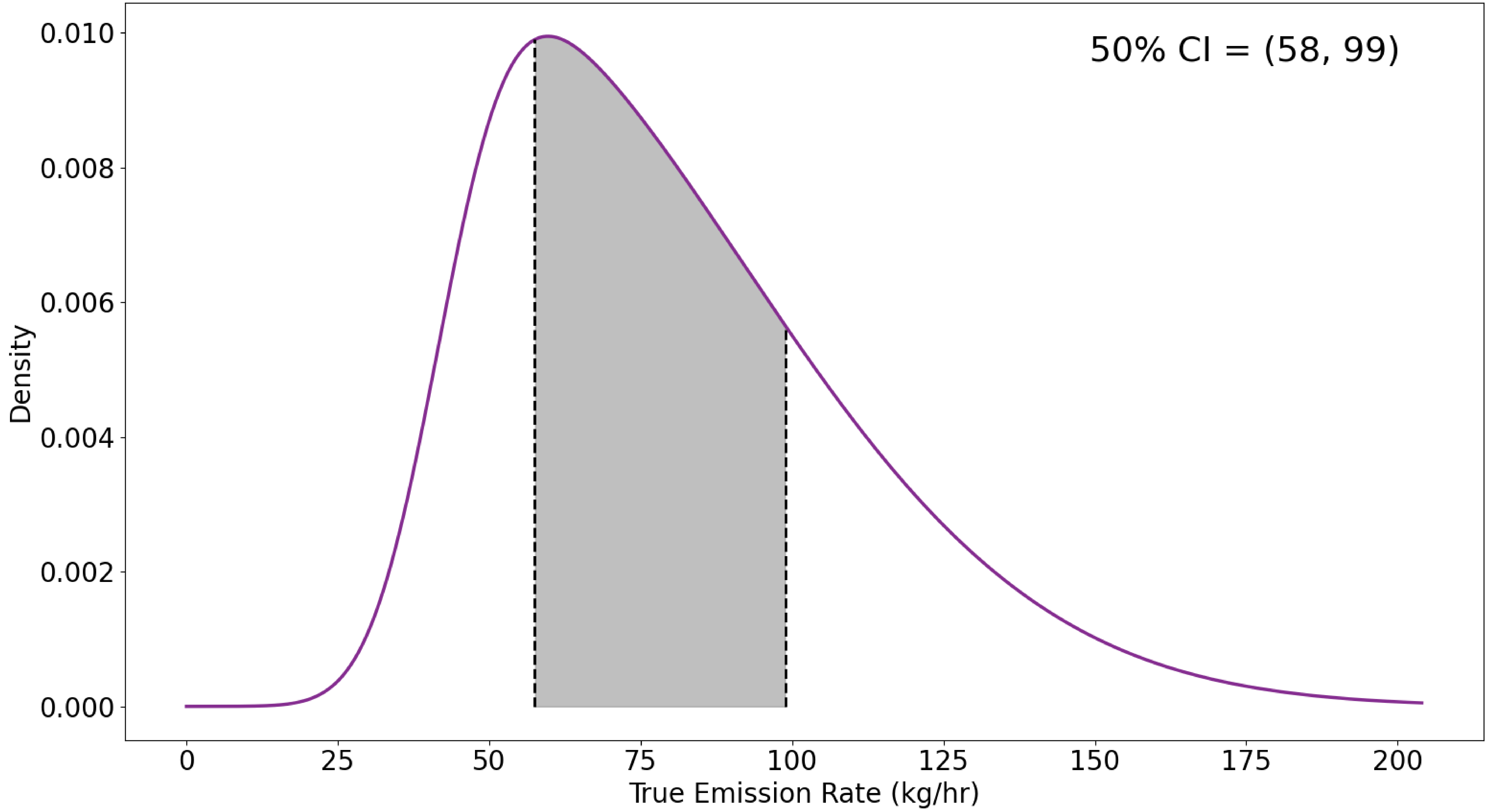
$$\text{Truth} = \text{Est.} \times \text{Est. Error} + \text{Est.}$$

Measured Emission Rate = 68 kg/hr



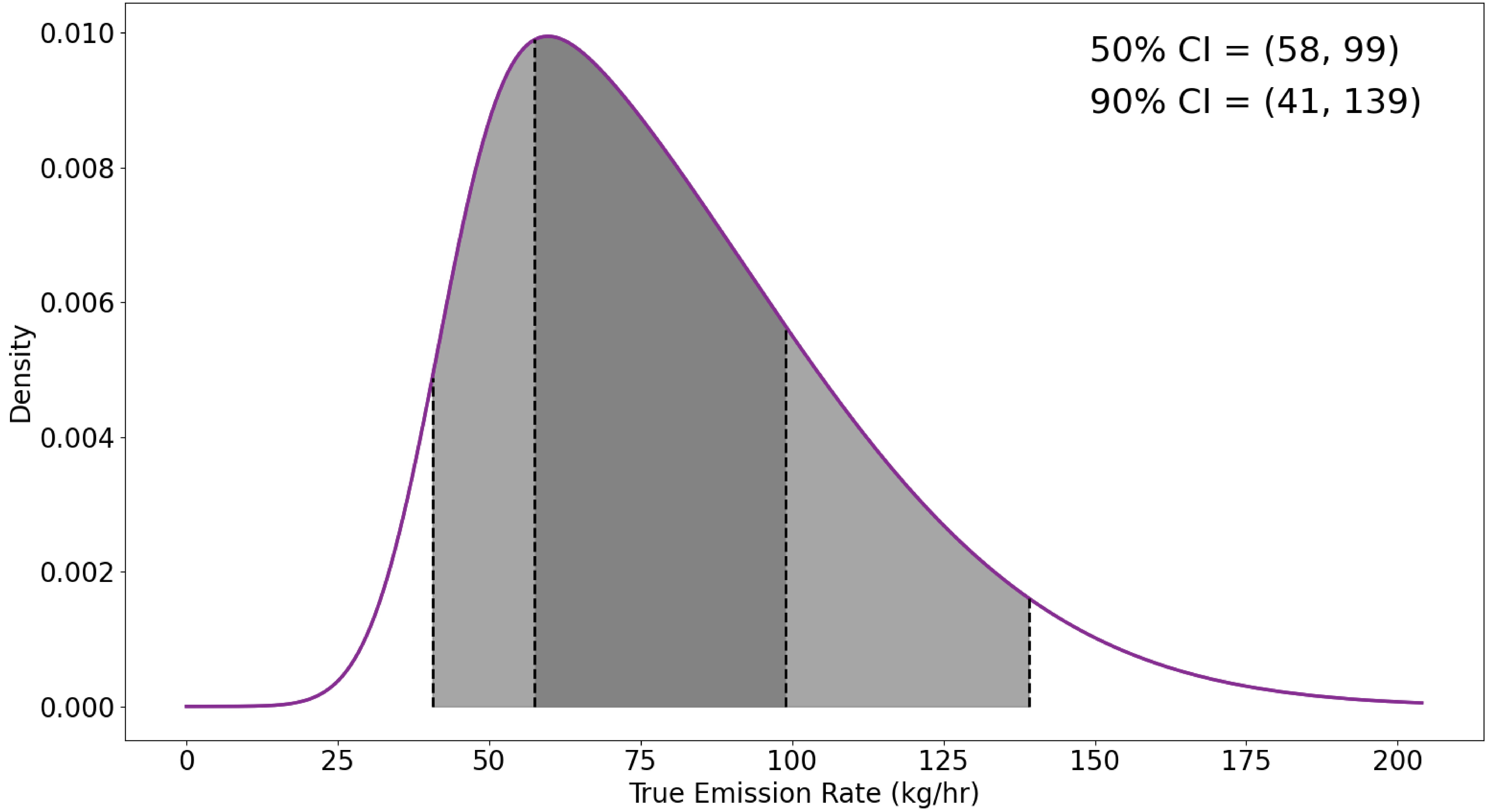
Use Example

Measured Emission Rate = 68 kg/hr



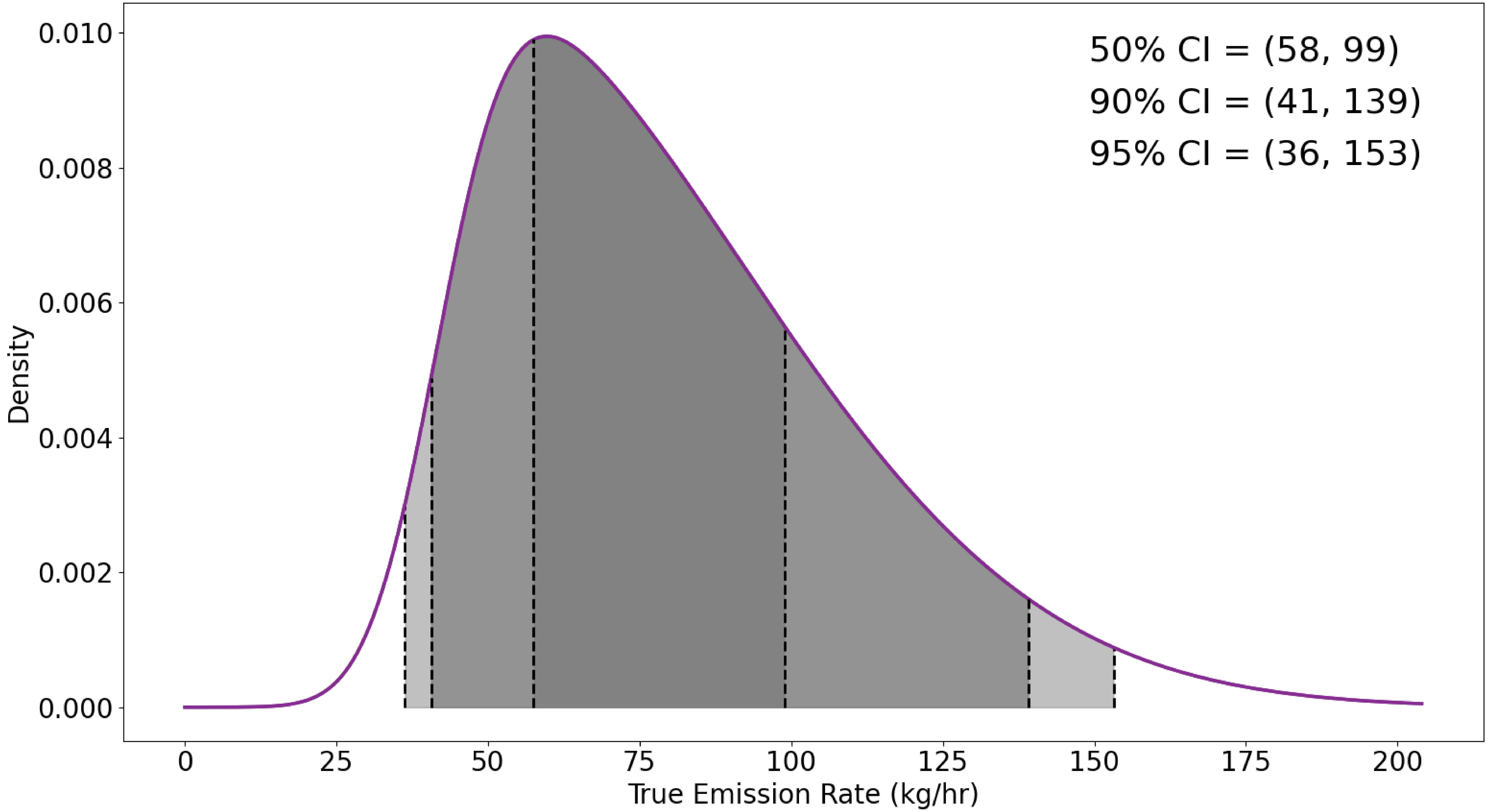
Use Example

Measured Emission Rate = 68 kg/hr



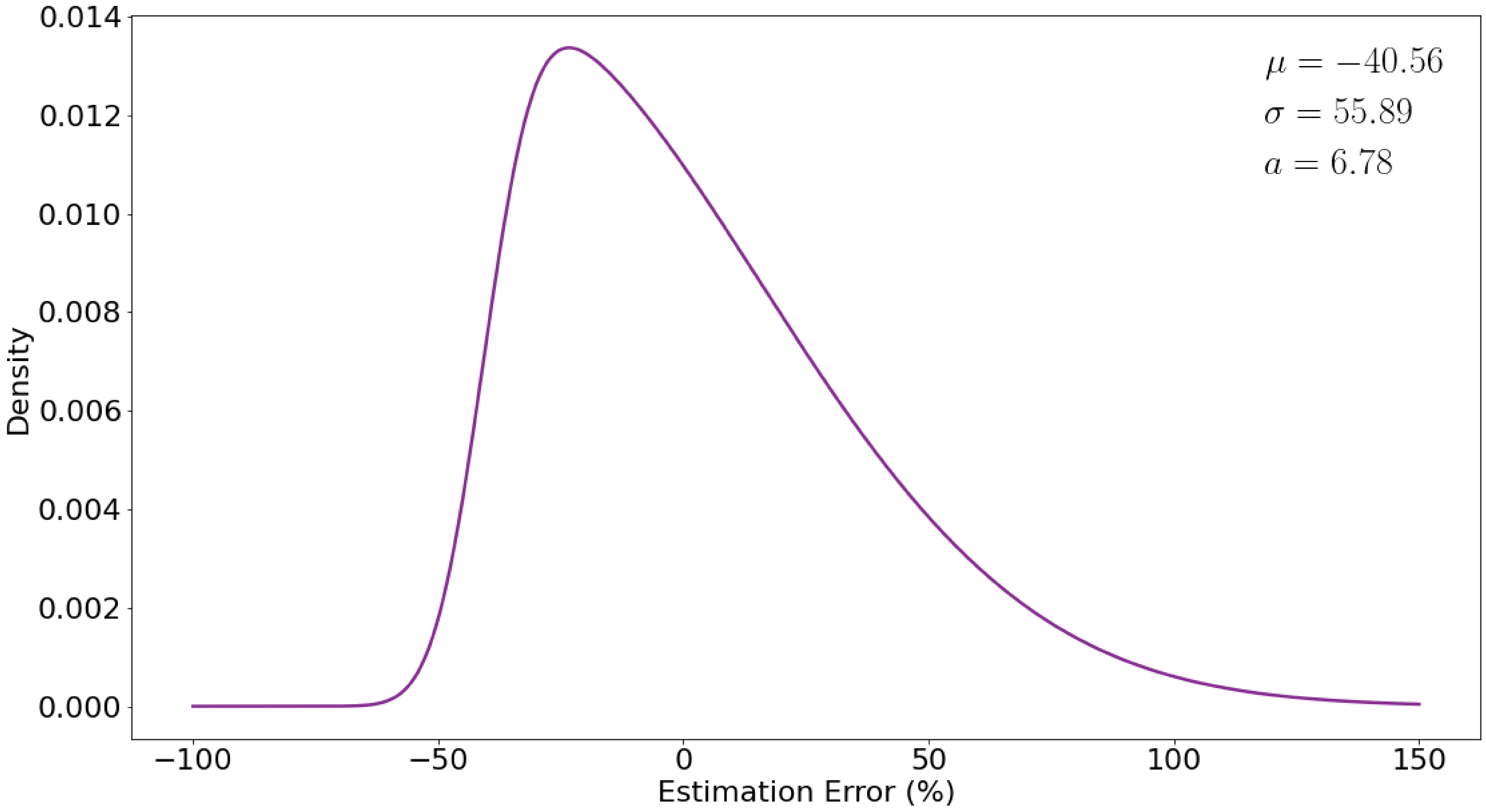
Use Example

Measured Emission Rate = 68 kg/hr



Use Example

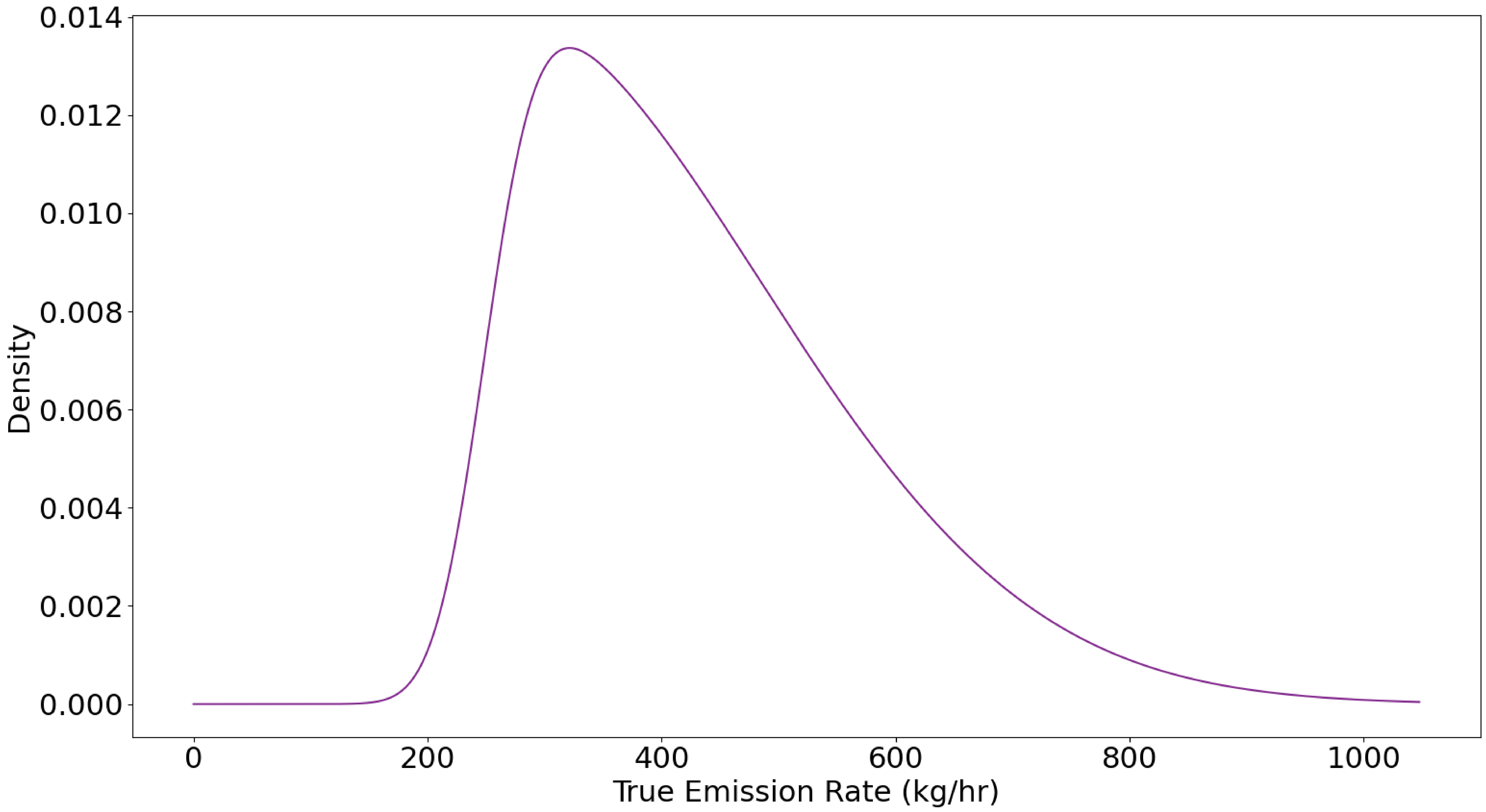
Measured Emission Rate = 419 kg/hr



Real-World Example

$$\text{Truth} = \text{Est.} \times \text{Est. Error} + \text{Est.}$$

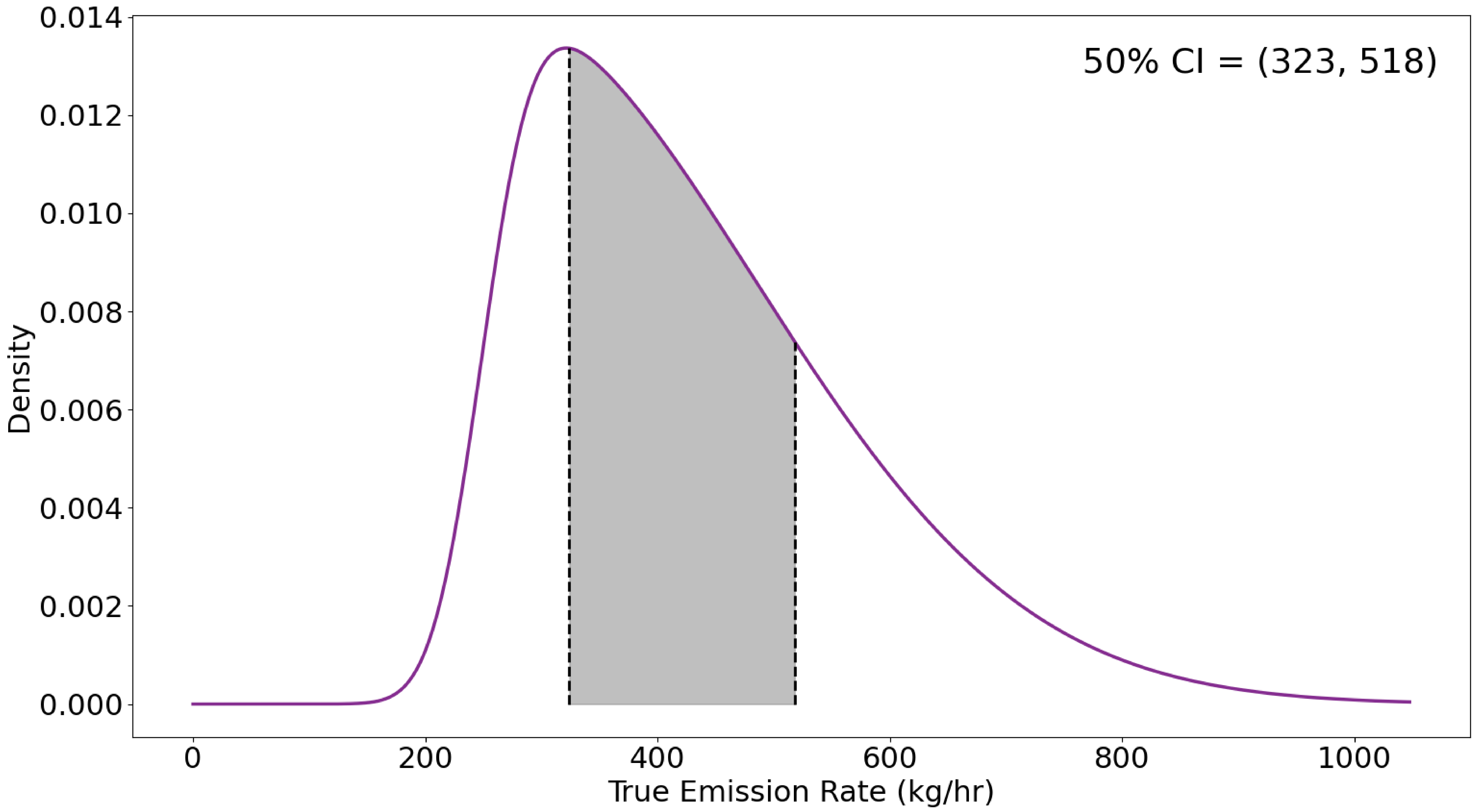
Measured Emission Rate = 419 kg/hr



COLORADO SCHOOL OF MINES
EARTH • ENERGY • ENVIRONMENT

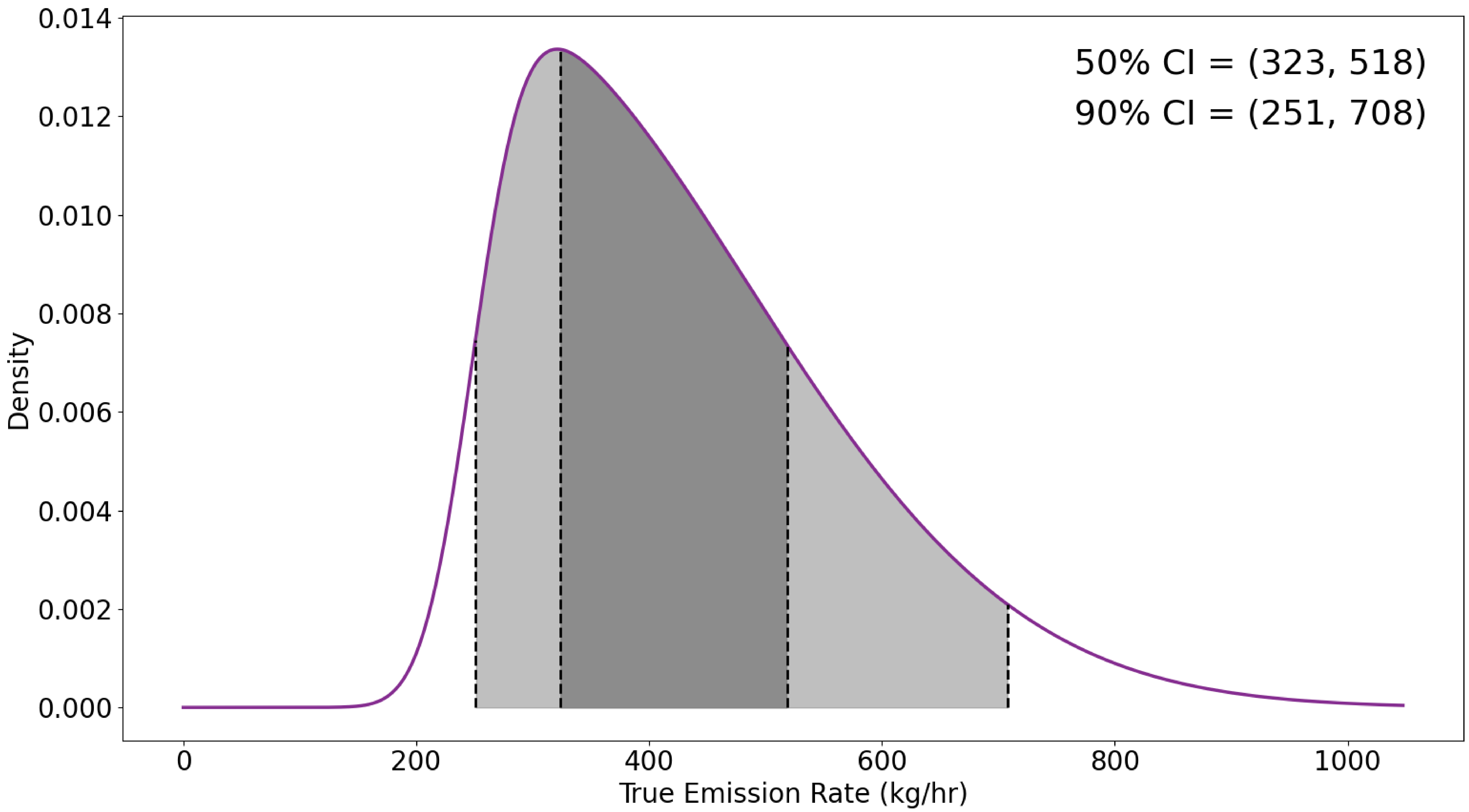
Use Example

Measured Emission Rate = 419 kg/hr



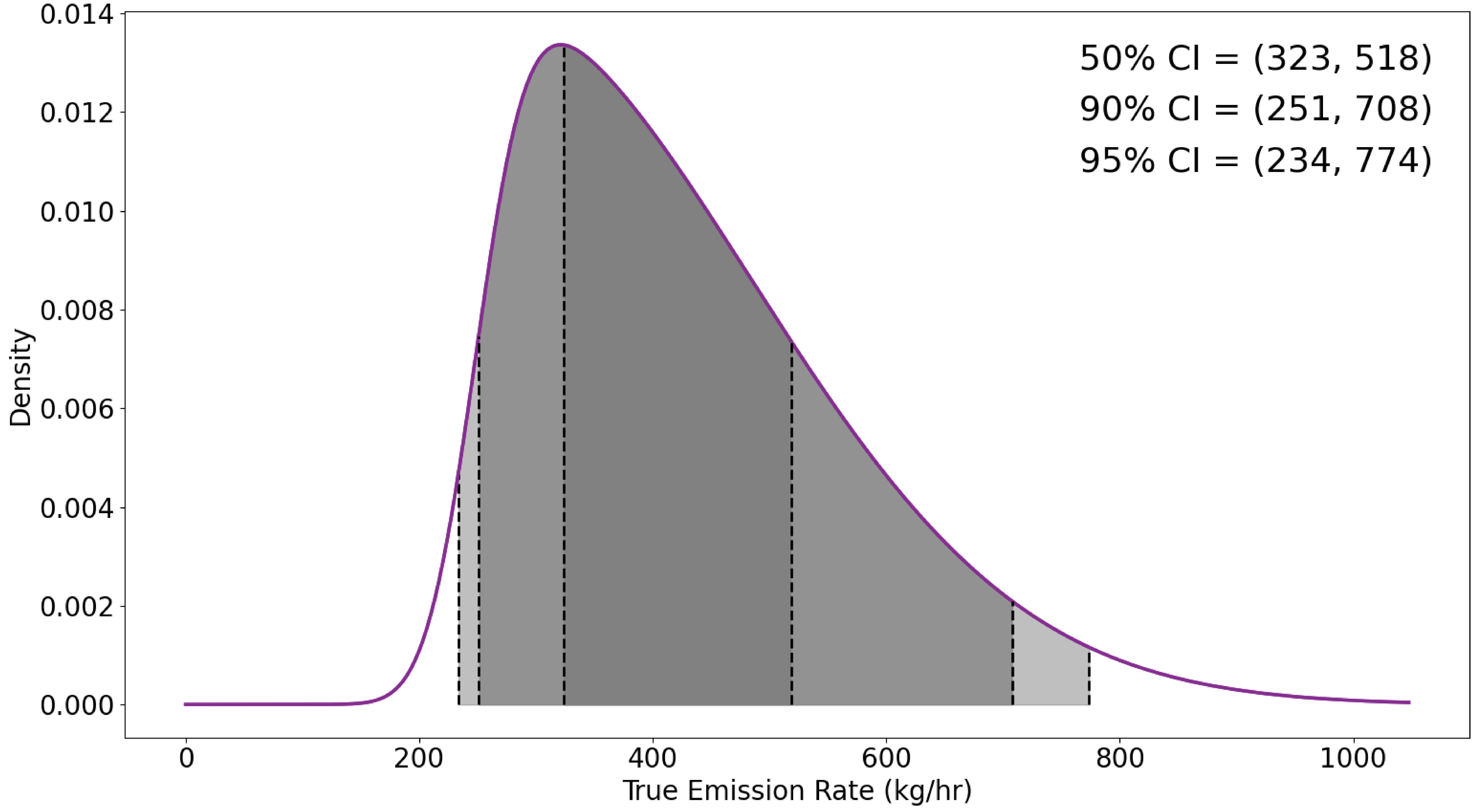
Use Example

Measured Emission Rate = 419 kg/hr



Use Example

Measured Emission Rate = 419 kg/hr



COLORADOSCHOOLOFMINES
EARTH • ENERGY • ENVIRONMENT

Implications

- Proposed framework generates a theoretical error distribution for any measured emission rate
  - Generates confidence intervals for individual measurements
  - Could help to construct error bounds for entire inventories
- Easily modifiable to use other predictors or multiple at once

drichardsdinger@mines.edu

Questions?

Thank you for attending!

COLORADO SCHOOL OF MINES
EARTH • ENERGY • ENVIRONMENT